

FIFTH PART

QUALITATIVE APPROACH

Answer to the question “ **HOW ?** ”

NUMBER OF DIMENSIONS OF AN OBJECT

- **THREE-DIMENSIONAL OBJECTS :**

Strictly speaking all material objects extend in a three-dimensional space.

Practically, this definition is restricted to objects irreducible to a smaller number of dimensions.

Example : 800.000 t lot of bauxite.

- **TWO-DIMENSIONAL OBJECTS** : when the third dimension, usually vertical, is ...
 - ◆ small as compared with the other two and ...
 - ◆ practically uniform.

Model : a sheet of paper.

Examples : a flat truck-load, a forest, a corn field, a flat mineral deposit, a « blister » copper plate etc.

● ONE-DIMENSIONAL OBJECTS

when the other two dimensions are :

- ◆ small as compared with the first
- ◆ practically uniform.

Model : cable.

Example : a batch of particulate solids, liquids or pulps **flowing** at a more or less uniform rate. A river. A rail.

Frequent. Easy and cheap to sample correctly.

● ZERO-DIMENSIONAL OBJECTS

By convention : a population made of a large number of units that can have :

◆ either a more or less uniform mass

Example : all manufactured objects.
Case dealt with by classical statistics.

◆ or different masses.

Example : mineral fragments whose mass can vary in a ratio of 1 to 10^{20} .
Case dealt with by Gy's model (1951).

CORRECT SAMPLING AND NUMBER OF DIMENSIONS

Experience shows that ...

- ◆ The larger the number of dimensions of an object, and **the more difficult and costly** its **correct sampling**.
- ◆ Correct sampling of irreducible three- or two-dimensional objects is ...

ECONOMICALLY UNACHIEVABLE

REDUCTION OF THE NUMBER OF DIMENSIONS OF AN OBJECT

- A CORRECT SOLUTION TO THE SAMPLING PROBLEM DOES EXIST ONLY **when it is** economically feasible to **reduce the number of dimensions** of a 3- or 2-dimensional object to 1- or 0-dimension, **which requires its reclaiming**

The economical feasibility obviously depends on the mass to be handled.

● REDUCTION FROM 3 / 2 to 1 dimension

The lot is reclaimed and transformed into **a flowing stream** :

◆ particulate solids : at the discharge of a conveyor belt,

◆ liquids and pulps : at the discharge of chutes or piping systems.

Example : pumping or gravity flow at the bottom of a three-dimensional tank.

- **REDUCTION FROM 3 / 2 to 0 dimension**
the lot is reclaimed and transformed into a population of usually ordered units :
 - ◆ **particulate solids** (trucks, waggons manual / mechanical shovelfuls, etc.)
 - ◆ **liquids and pulps of ground solids in a liquid** (drums, containers, etc .)

Example (world record, Japan): 16.000 t lot of valuable mineral, reduced to an eight-ton sample processed in the lab.

QUALITATIVE APPROACH

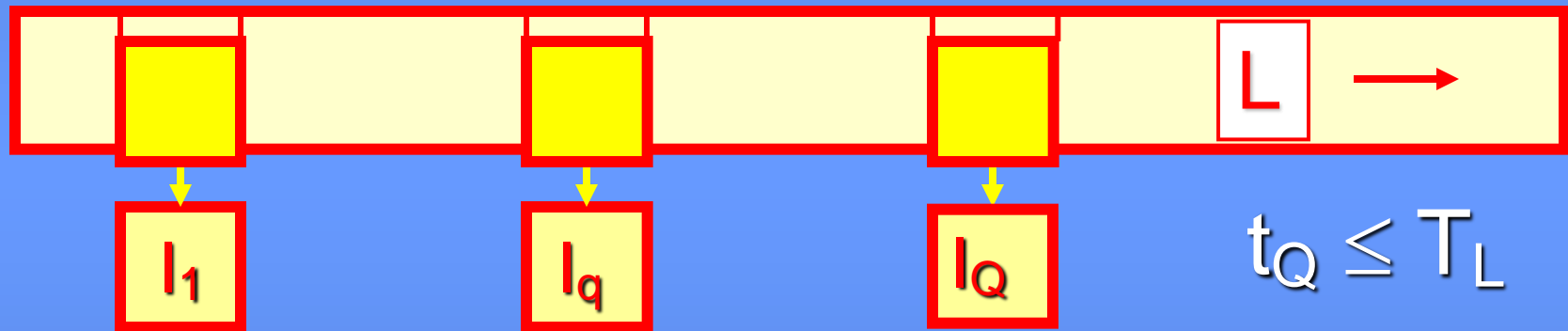
HOW TO ACHIEVE A MASS REDUCTION

FIRST MASS REDUCTION MODEL

- INCREMENTAL SAMPLING

of a one-dimensional flowing lot :

Lot L flows from instants $t = 0$ to $t = T_L$

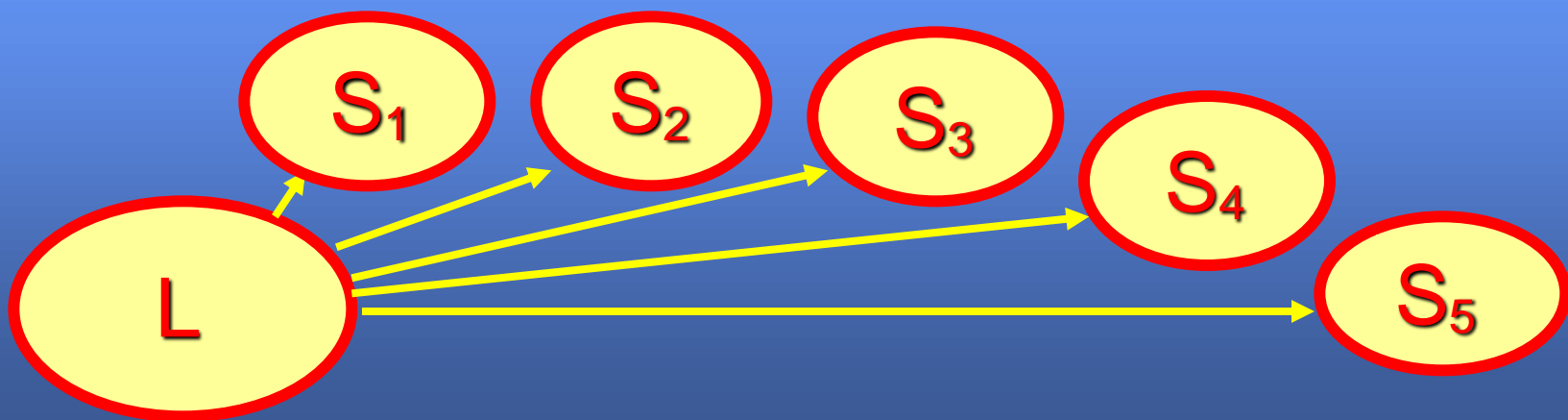


At instants $t = t_1$, $t = t_2$, $t = t_q$, $t = t_Q$
 Q point-increments are selected and
extended into Q material-increments I_q

Material Sample S is : $S \equiv \sum I_q \ll L$

SECOND MASS REDUCTION MODEL

- **SPLITTING** of a zero-dimensional lot.
Lot **L** is split between **N** twin-fractions that will become **N** twin potential sub-samples. Sample **S** is obtained by selecting and gathering one or several potential sub-samples $S_k : L \equiv \sum_k S_k$



QUALITATIVE APPROACH

THE SAMPLING OF ZERO-DIMENSIONAL MATERIAL BATCHES

CORRECT SAMPLING OF A ZERO-DIMENSIONAL LOT

Lot L is made of N unspecified units U_n the mass of which may be uniform or not.

$n = 1, 2, \dots, N$ ♦ N is large : $N \gg 1$

Q units I_q are selected : $1 \ll Q \ll N$.

No hypothesis re. a possible correlation between n and properties of U_n is made.

Sample $S = \sum_q I_q$. Correctness is achieved by introducing a random factor.

LOT MADE OF N WELL-DEFINED UNITS OF COMPARABLE MASS

Three selection modes : SY , ST , RA

- SY : Systematic sampling with uniform interval N_{sy} .

Random selection of increment l_1 in the first interval $1 \leq l_1 < N_{sy}$.

Then, increment l_q is defined by :

$$l_q = l_1 + (q - 1) N_{sy} \leq N$$

- **ST** : Stratified random.

The lot L is first divided into Q strata T_q of equal length T_{st} . An increment I_q is selected at random within each stratum T_q

- **RA** : Random.

Q values of q are selected at random between 1 and N , which defines Q increments I_q

- **SAMPLE**. In the three cases, sample S is the sum of the Q increments I_q

SY : L \equiv 756 DRUMS OF RADIOACTIVE RESIDUES ♦ COMPARABLE MASS



1

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Lot = 63 groups of 12 drums. Selection of
n°4 at random. Interval = 12. S = n° 4, 16, 28

ST : L \equiv 756 DRUMS OF RADIOACTIVE RESIDUES ♦ COMPARABLE MASS

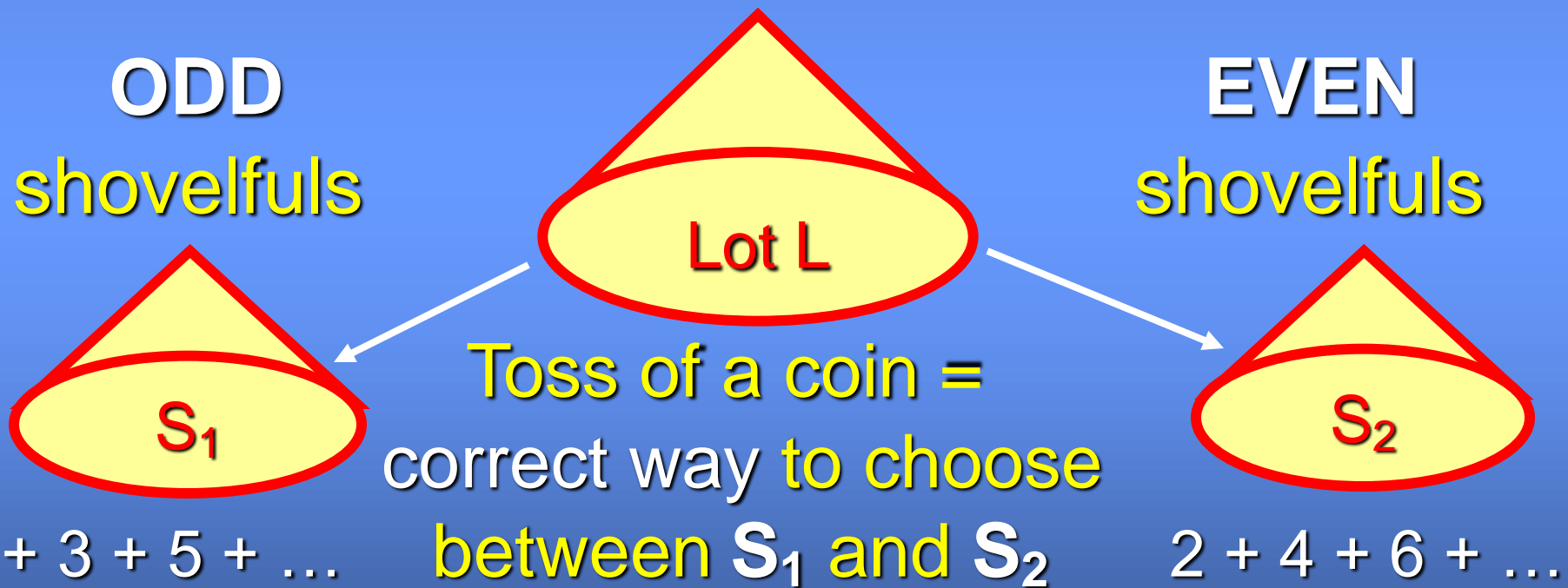
											
1	2	3	4	5	6	7	8	9	10	11	12
											
13	14	15	16	17	18	19	20	21	22	23	24
											
25	26	27	28	29	30	31	32	33	34	35	36

Lot = 63 strata of 12 drums. S = random selection of n° 4, 23, 30 in strata 1, 2, 3.

SPLITTING **L** INTO TWIN-SAMPLES

◆ ALTERNATE SHOVELING ◆

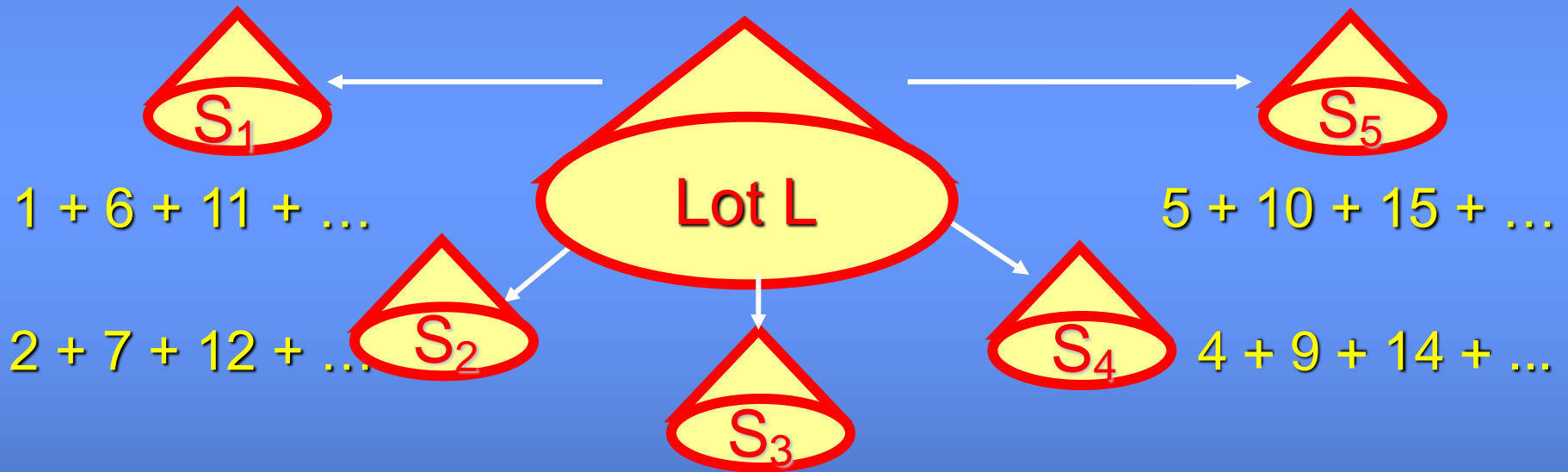
Lot **L** split by alternate shoveling into **S₁** and **S₂** = twin potential samples



The operation is usually repetitive

EQUAL FRACTIONAL SHOVELING

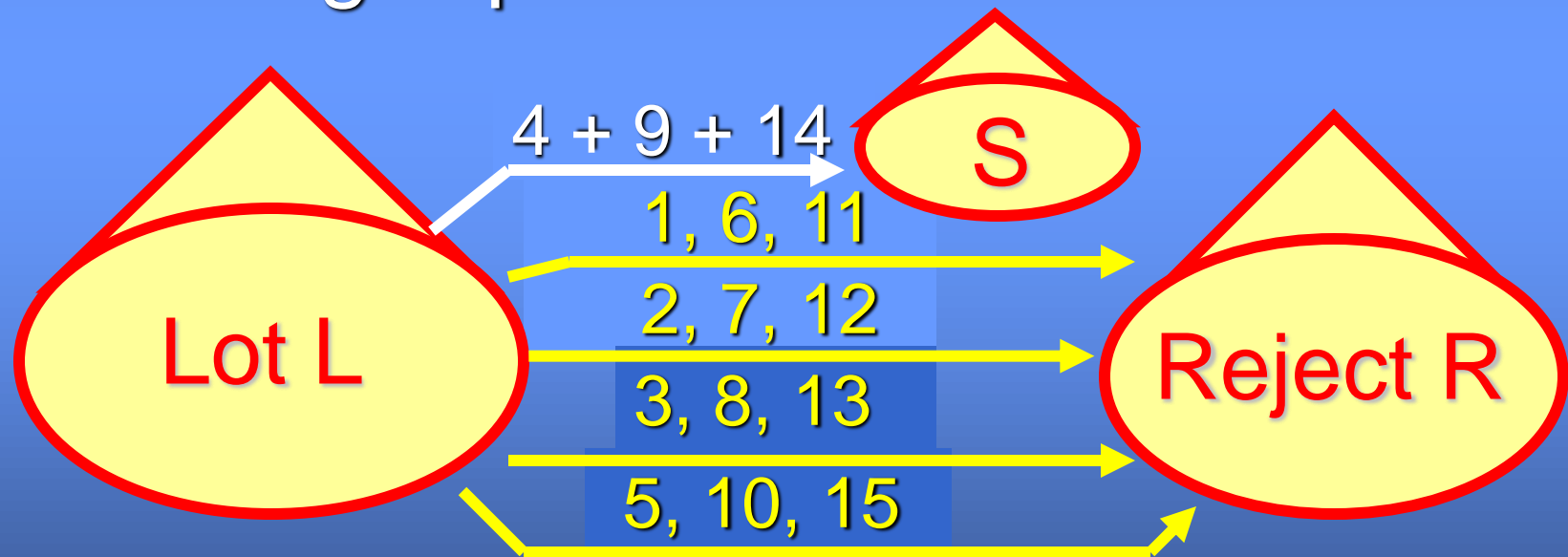
L split by **EQUAL** fractional shoveling
between **N** twin-samples. Rate $1/N = 1/5$



Correct if actual
sample selected at random : S_1 / S_5

UNEQUAL FRACTIONAL SHOVELING

L is distributed between two non-twin fractions by **UNEQUAL** fractional shoveling. This method is **NON-PROBABILIST**. Cheating is possible but can be avoided.



Japan : 16.000 t to 8 t : 1/20 ; 1/10 ; 1/10

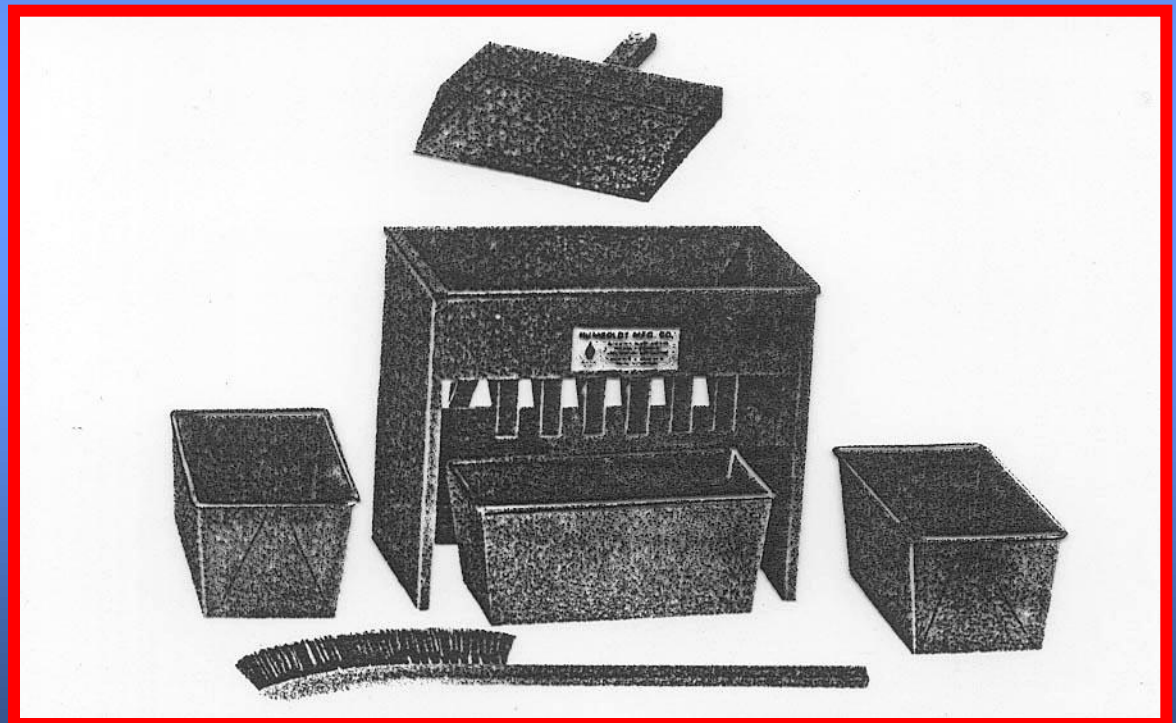
THE RIFFLE SPLITTER

Laboratory device meant to split into two « halves » lots weighing (in 2000) up to 1000 kg. Openings from 6 to 80 mm.

$d_{\max} < 2,5$
to 30 mm

Repetitive
operation

Commercial
sampling



THE RIFFLE SPLITTER

Laboratory device. **Reliable when implemented correctly (see below).**

Number N of chutes of a riffle splitter :

$$12 \leq N \leq 24 \text{ with } N \text{ even}$$

Chute width $W = 6$ to 90 mm.

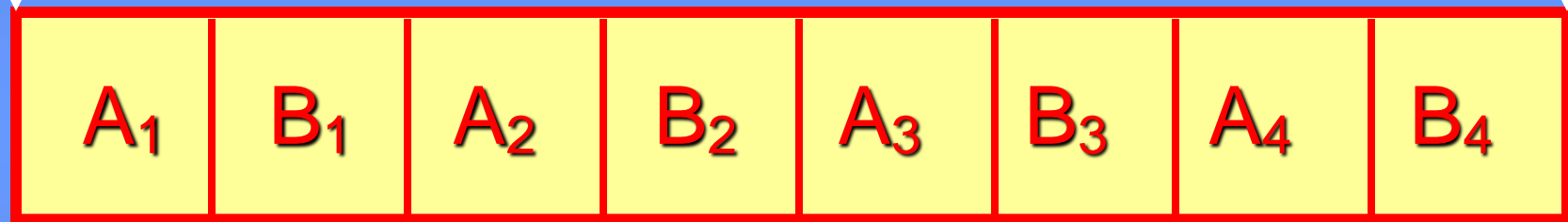
To prevent obstruction : $W \geq 3 \times d$ (fragment diameter). Practically: $2 < d < 30$ mm

**RIFFLE SPLITTERS ARE USUALLY
CORRECTLY DESIGNED**

Illustrations : Courtesy of Sepor, California

PRINCIPLE OF RIFFLE SPLITTING

**ZERO-DIMENSIONAL LOT L SPREAD
ON A RECTANGULAR SHOVEL**

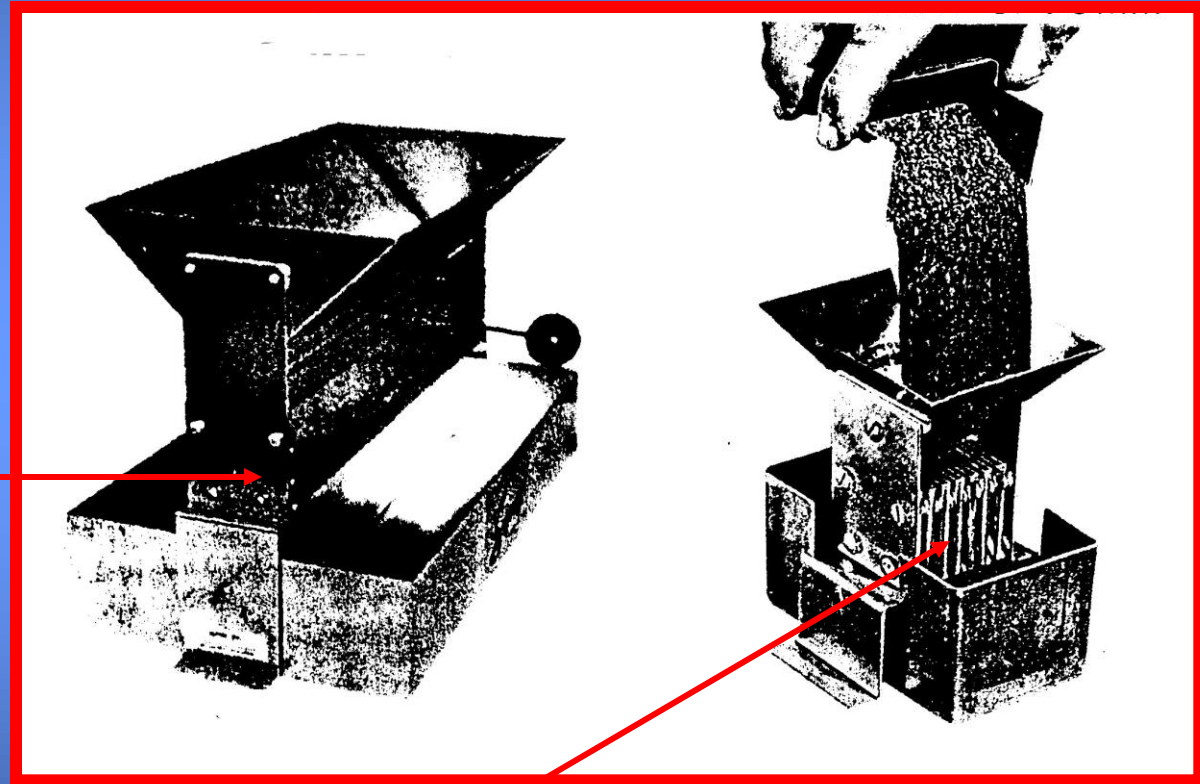


$$\begin{array}{ccccccc} \boxed{A_1} & + & \boxed{A_2} & + & \boxed{A_3} & + & \boxed{A_4} & = S_A \\ & & \boxed{B_1} & + & \boxed{B_2} & + & \boxed{B_3} & + & \boxed{B_4} & = S_B \end{array}$$

Correct selection of S_A or S_B by toss of a coin.

VARIOUS MODELS OF RIFFLE SPLITTERS

Hopper with gate : **allows mixing prior to splitting.**



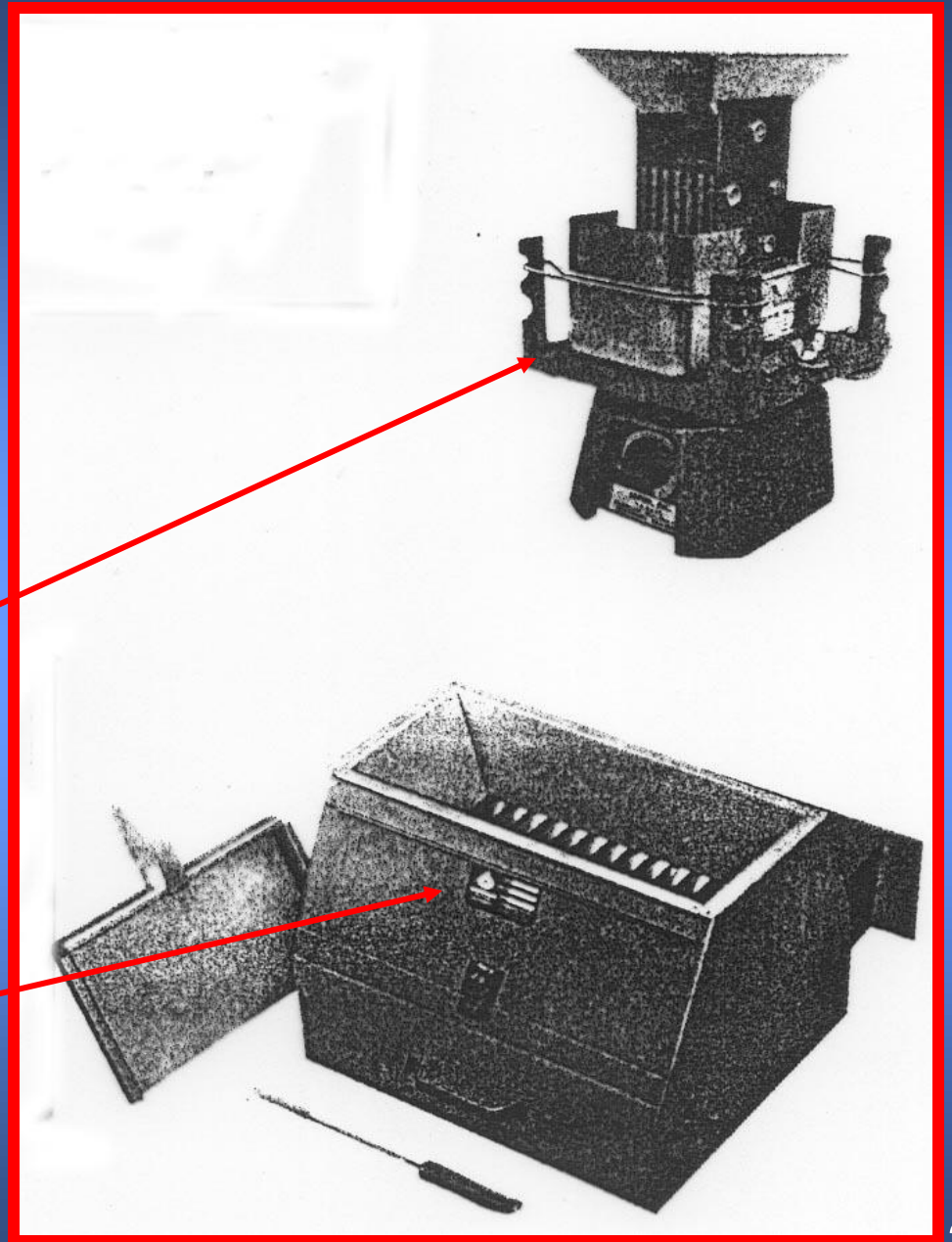
Micro-splitter : **chute width 3 to 10 mm**

RIFFLE SPLITTERS

Dust preventing
systems

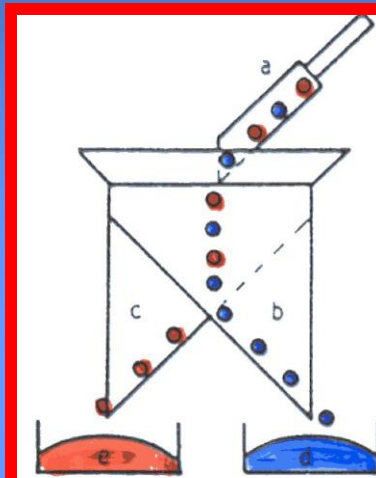
Vibrating

Enclosed

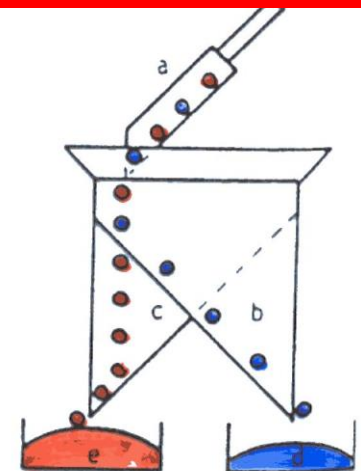


INCORRECT USE OF A CORRECT RIFFLE SPLITTER

Experiment
carried out
in 1952



Correct



Incorrect

Four splitting stages → 16 samples
observed biases : **G = left ; D = Right**

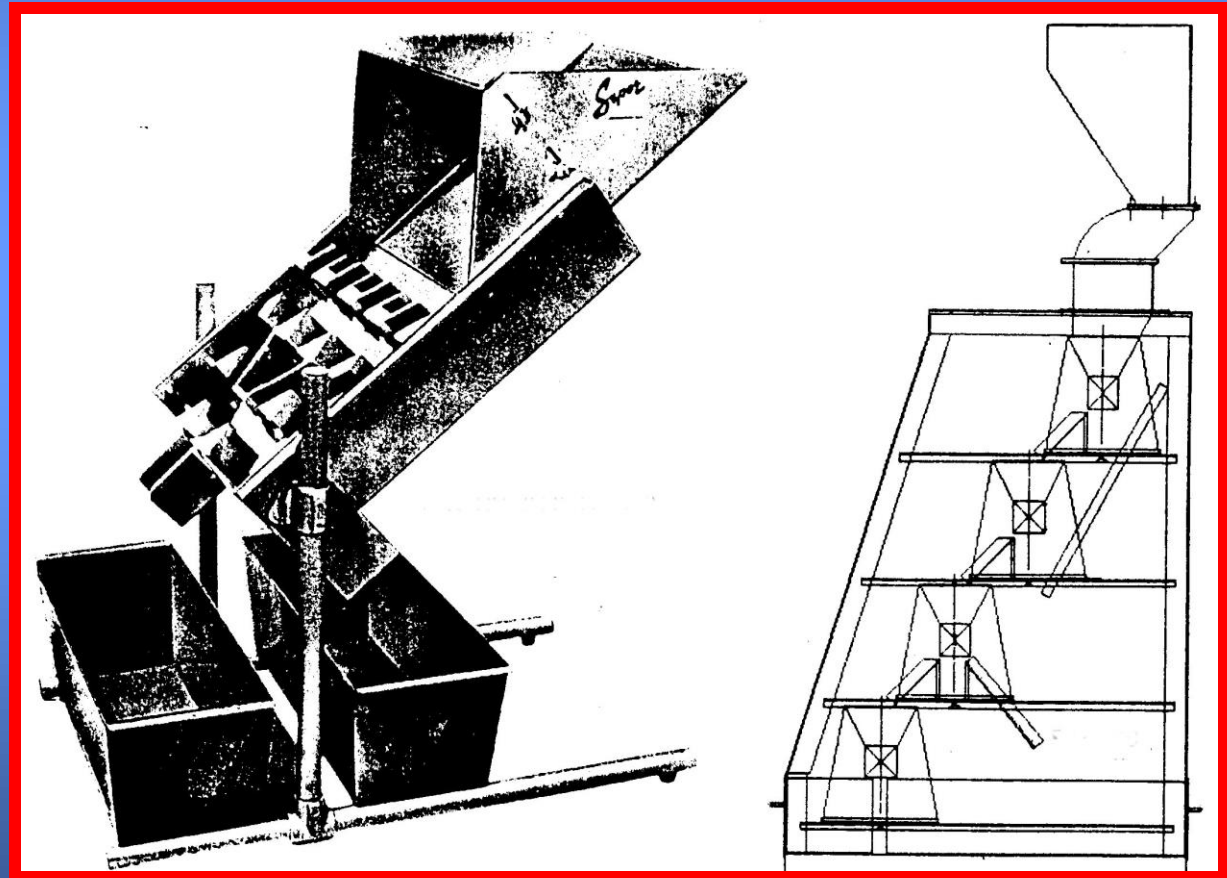
Mass : G > D (6%) ♦ Pb : D > G (2%)

4 x G 0 x D	3 x G 1 x D	2 x G 2 x D	1 x G 3 x D	0 x G 4 x D
GGGG - 0,201	GGGD - 0,051	GGDD + 0,269	GDDD + 0,169	DDDD + 0,214
	GGDG - 0,221	GDGD + 0,174	DGDD + 0,139	
	GDGG - 0,341	GDDG + 0,084	DDGD + 0,094	
	DGGG - 0,281	DGGD - 0,251	DDDG + 0,109	
		DGDG - 0,101		
Negative bias	Negative bias	DDGG + 0,189	Positive bias	Positive bias

CASCADE OF RIFFLE SPLITTERS

Designed to extract $1/16$ in a single pass.
This device is **INCORRECT** in its principle

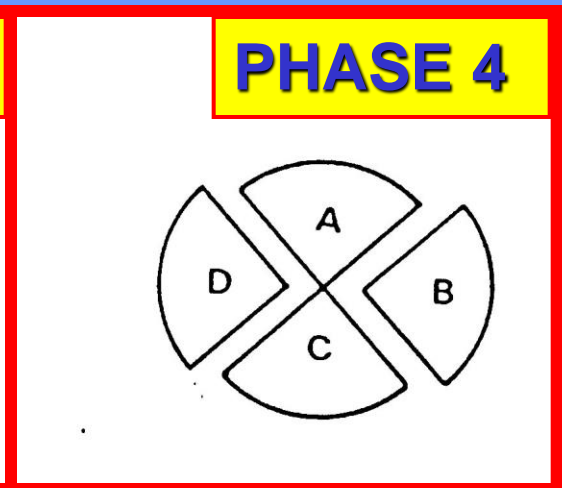
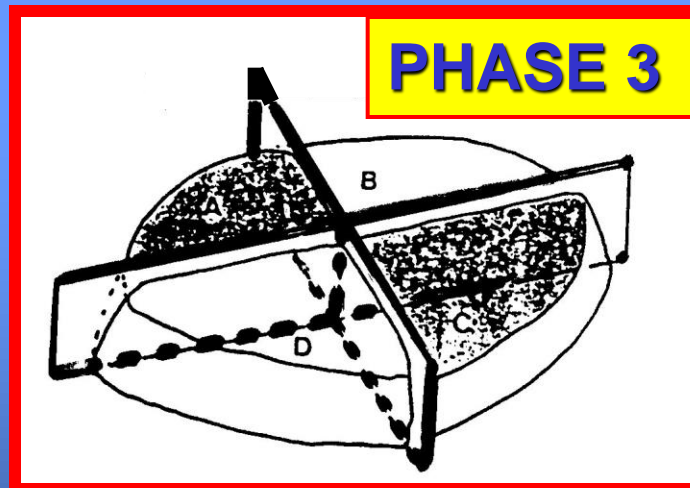
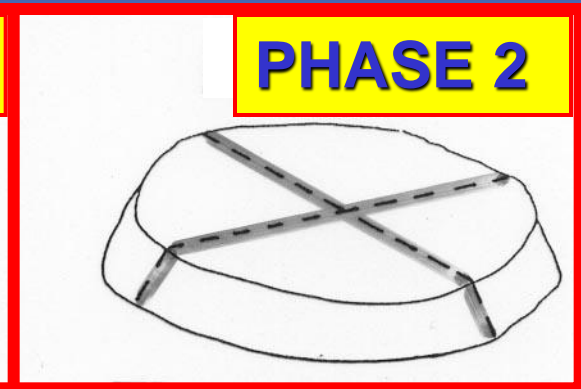
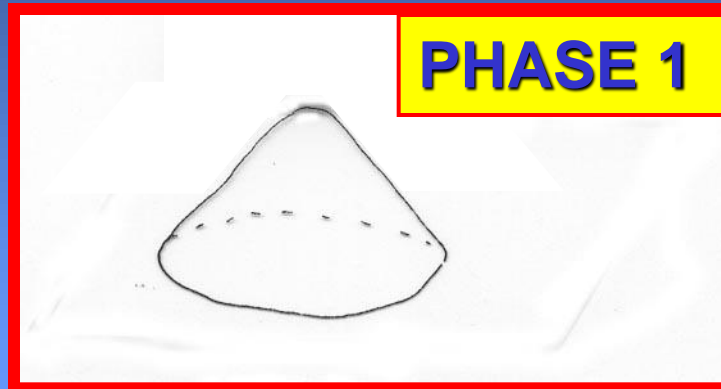
SHOULD BE
AVOIDED :
SAMPLES
ARE
BIASED



CONING AND QUARTERING

Uselessly
costly
hand
method

Alternate
shoveling
always
better

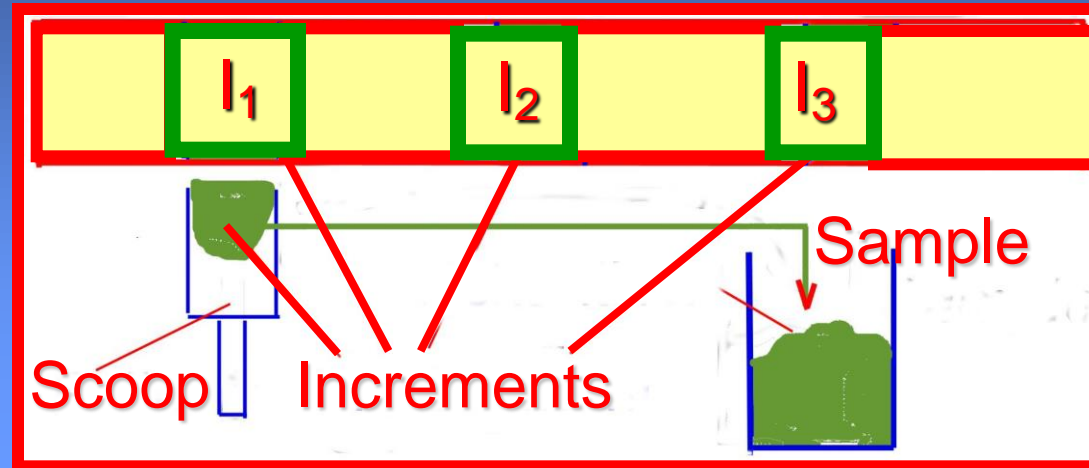


Two potential samples (A+C) or (B+D)

THE JAPANESE SLABCAKES

ONE-
DIMENSIONAL

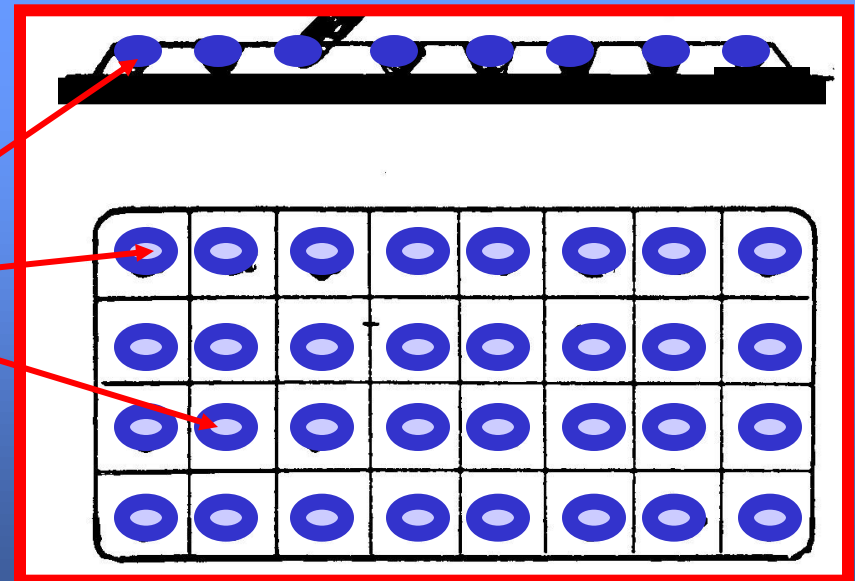
Possibly biased



TWO-
DIMENSIONAL

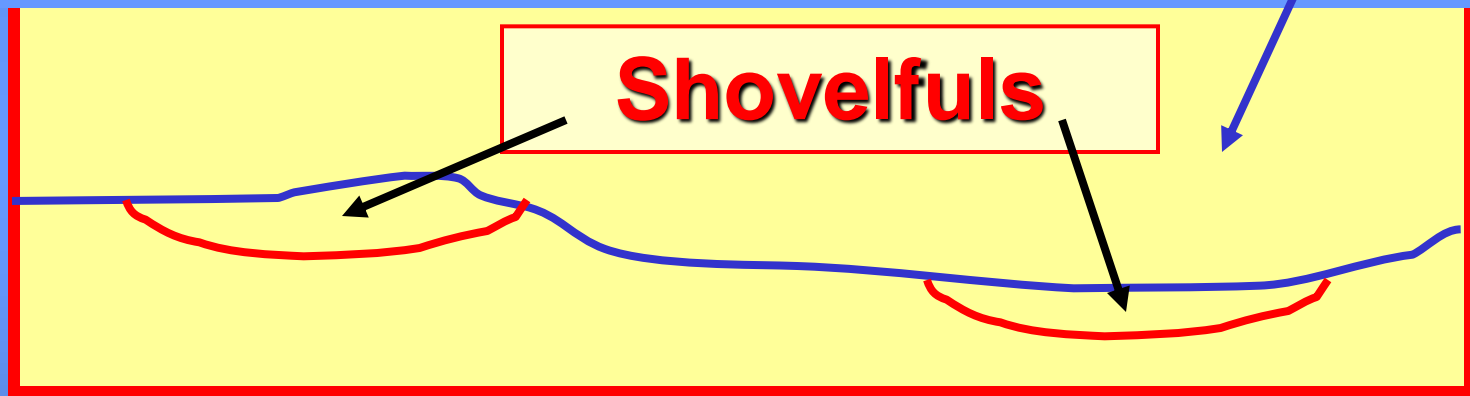
Increments

Very popular in the
Pacific Area



Example : Cattle Food “SAMPLING” =
NON-PROBABILIST Picking by means
of a shovel

Any container



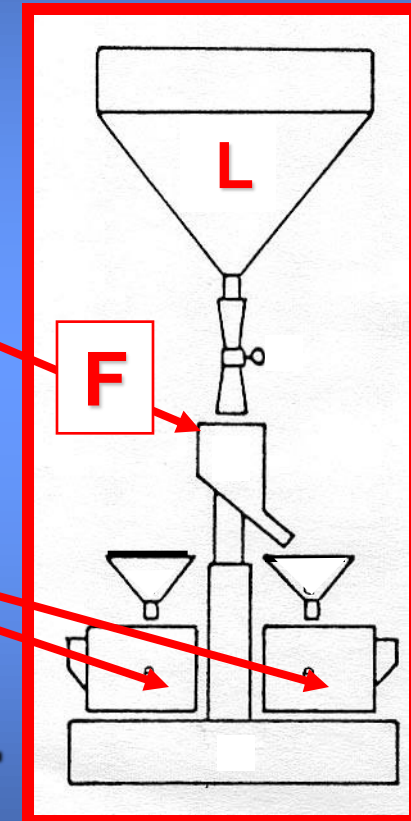
One shovelful = one increment
Specimen = Σ increments

SECTORIAL DIVIDER WITH A REVOLVING FEEDER

Usually correct **laboratory** but also **pilot plant device**. The feeder **F** revolves about its axis.

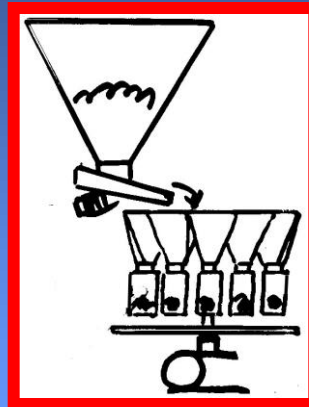
Irrespective of its mass, lot **L** can be divided into **N** twin-fractions. Each of these can be regarded as a $1/N$ th sample of **L**.

Typically : **N = 6, 8, 12**. Commercial.

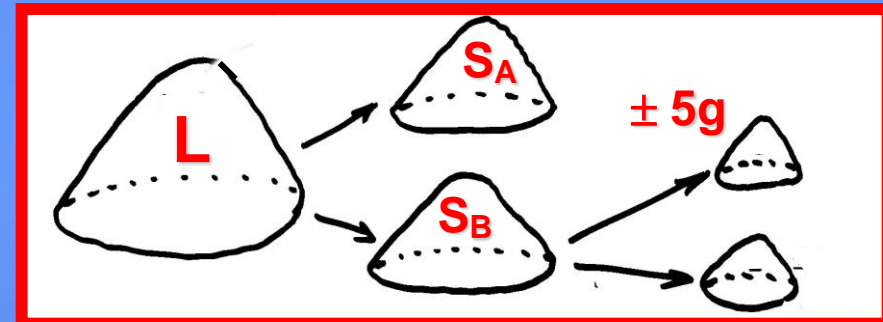


OBTENTION OF A TEST-PORTION

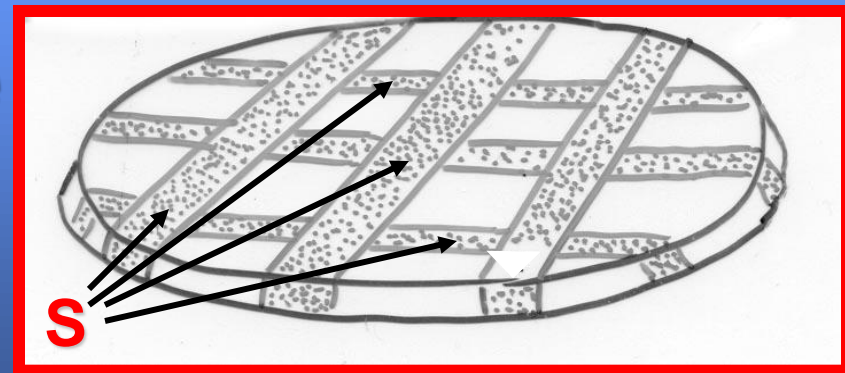
- Use of a rotary divider to obtain a 50-1000 g laboratory sample. This is a one-dimensional sampling.



- Alternate shoveling by means of a scoop to obtain 5-50 g



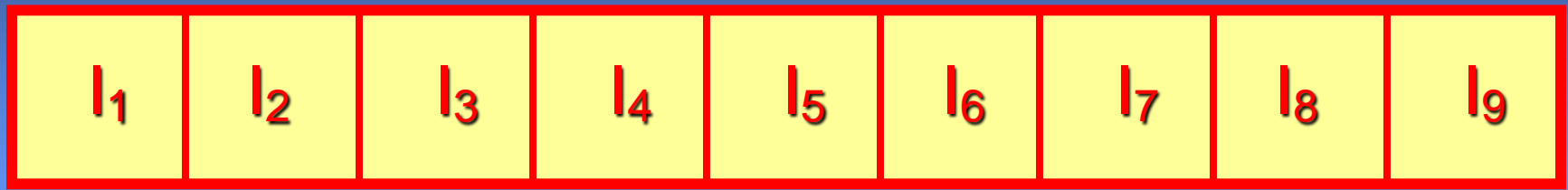
- Mixing and taking the test-portion (e.g. 1 g) by means of a spatula.



QUALITATIVE APPROACH

THE SAMPLING OF ONE-DIMENSIONAL MATERIAL BATCHES

PROPERTIES OF A ONE-DIMENSIONAL OBJECT



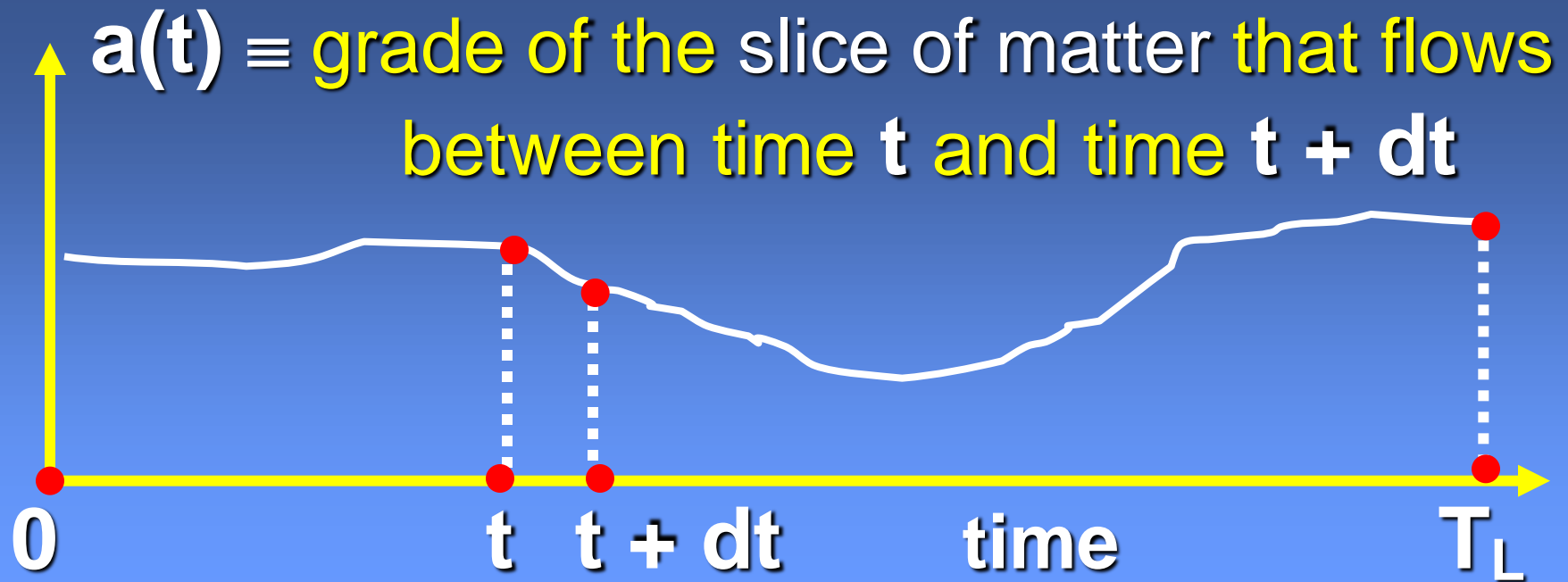
A one-dimensional, **compact or flowing** object **can be regarded** as a series of potential increments such as $l_1 \dots l_9$. **Each increment l_q may be regarded** as a zero-dimensional object, as autocorrelation **is barely perceptible on a small scale**. Residual autocorrelation generates a **0-dim. Grouping and Segregation Error GSE**

AUTOCORRELATION OF A SERIES

Correlation between two units of a series is called « autocorrelation of the series ». This is the correlation between the properties of increments I_q (taken at instant t_q) and I_{q+j} (taken at instant t_{q+j}) of this series. It is a function of the distance jT_0 (T_0 being the uniform interval).

This autocorrelation gives the series its dimension. Difference with a population.

It is represented by a quasi-continuous curve $a(t)$...



Correlation is sensible only in the long range.

On the scale of an increment of length dt or Δt it is negligible. For all practical purposes such an increment can be regarded as a zero-dimensional object.

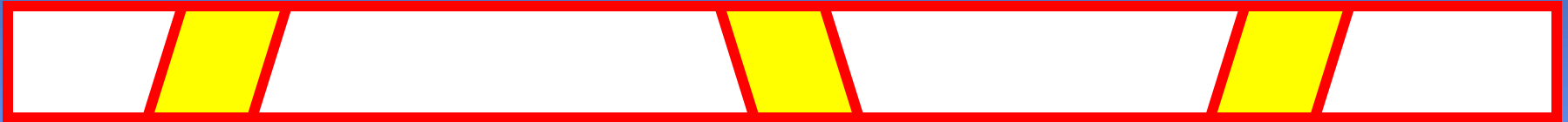
THE SAMPLING OF ONE-DIMENSIONAL FLOWING LOTS

Particulate solids, liquids, gases, fumes

Three ways to reduce their mass :

- 1** Taking the totality of the stream during a fraction of the flowing time,
- 2** Taking a fraction of the stream during the totality of the flowing time,
- 3** Taking a fraction of the stream during a fraction of the flowing time.

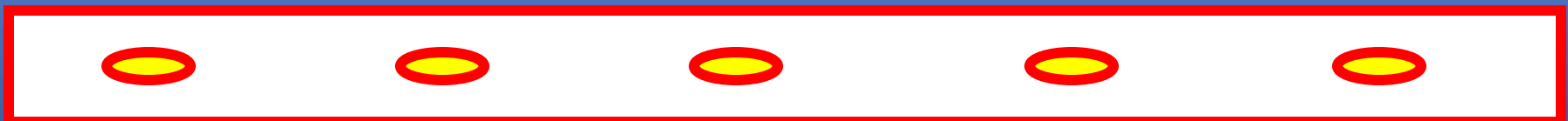
1 Taking the **TOTALITY** of the stream during a **FRACTION** of the time,



2 Taking a **FRACTION** of the stream during the **TOTALITY** of the time,

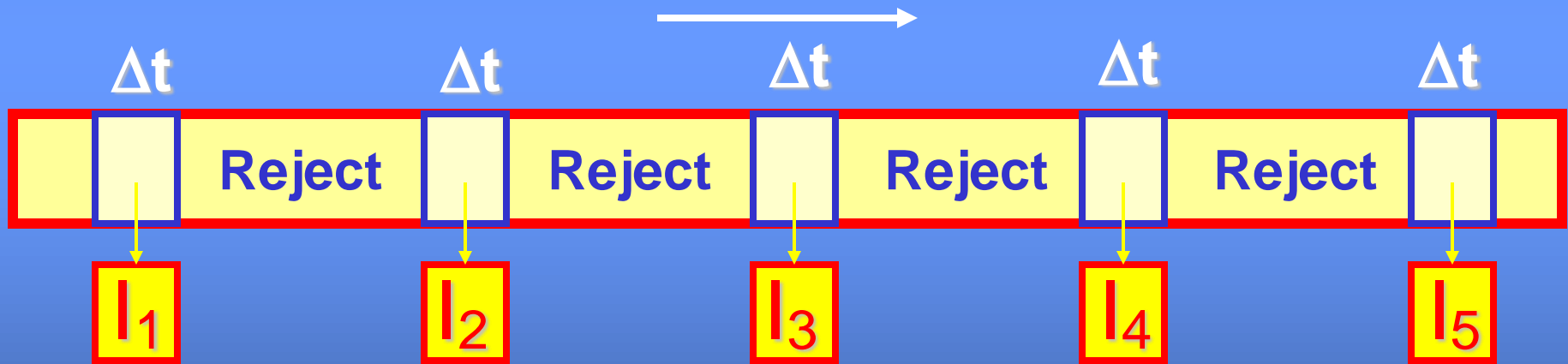


3 Taking a **FRACTION** of the stream during a **FRACTION** of the time,



1. TOTALITY OF THE STREAM DURING A FRACTION OF THE TIME

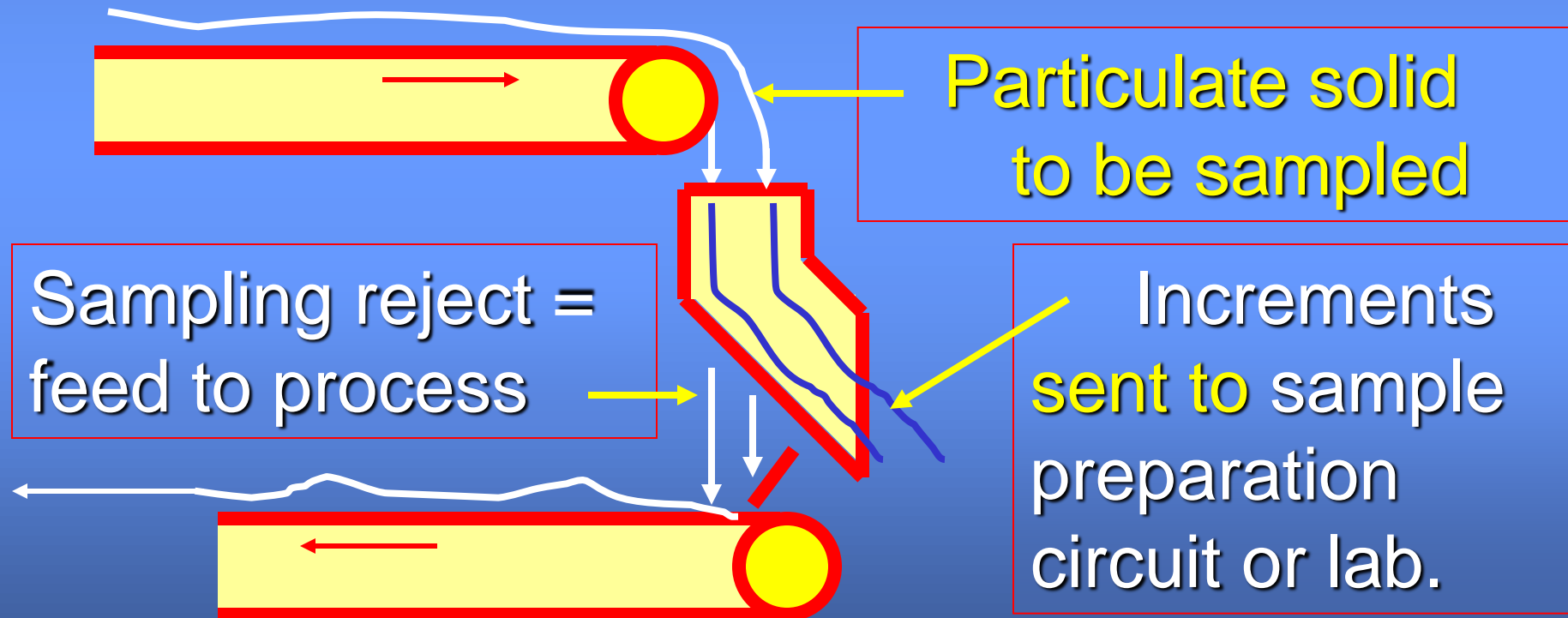
This can be achieved by cutting the stream at a uniform interval T_{sy} during a certain lapse of time Δt . Principle :



Increments I_q ♦ Sample $S = \sum_q I_q$
Sampling Reject = Main stream

CROSS-STREAM SAMPLERS

Extract the totality of the stream during a **FRACTION** of the time. Operate at discharge of belt conveyor or piping system.



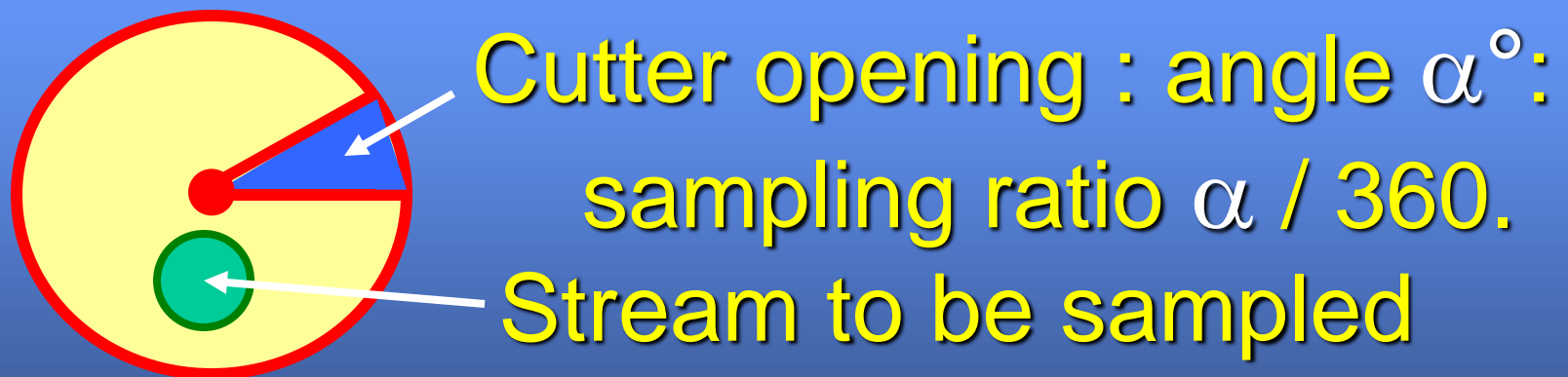
CROSS-STREAM SAMPLERS

- **STRAIGHT-PATH CUTTER :**



Moves between idle positions **G** and **D**.

- **CIRCULAR-PATH CUTTER :**



CROSS-STREAM SAMPLING IS THE

**ONLY PROBABILIST
SAMPLING METHOD**

**OF ONE-DIMENSIONAL OBJECTS.
FURTHERMORE ...**

**IT CAN EASILY BE
RENDERED CORRECT**

2 FRACTION OF THE STREAM DURING TOTALITY OF THE TIME

This method is **NON-PROBABILIST**

S →



If there is a **correlation** between the properties of a constituent and its position in the stream cross-section (e.g. gravity segregation) the specimen **S** may be heavily biased. Often used in chemical and pharmaceutical industries. 45

Implemented in chemical industries

LOT L TO BE SAMPLED



The diagram illustrates a sampling process in a tank. A large yellow rectangular box at the top contains the text 'LOT L TO BE SAMPLED' in blue, with a red arrow pointing to the right. A red line descends from the bottom of this box, forming a U-shape that leads into a vertical tube. This tube extends down into a tank. The tank is divided into two horizontal sections: a top blue section and a bottom yellow section. A red arrow points from the text 'Unreliable Specimen' to the yellow section of the tank. The entire diagram is set against a blue background.

Hypothesis
of homogeneity
throughout the
cross-section.

**Unreliable
Specimen**

Practically never observed.

NEARLY ALWAYS BIASED

3 FRACTION OF THE STREAM DURING FRACTION OF THE TIME

This method is **NON-PROBABILIST**

 l_1  l_2  l_3  l_4  l_5

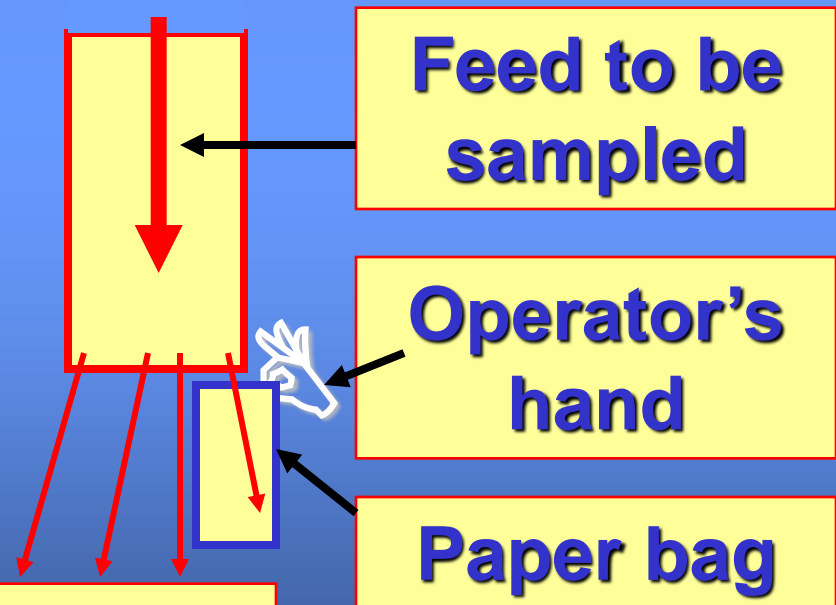
Examples : One shovelful every now and then on top of a belt load. Responsible for a \$7 million cheating over 3 years (tin concentrates). Also used in chemical and pharmaceutical industries.

FRACTION OF THE STREAM DURING FRACTION OF THE TIME

Example : Cattle Food “SAMPLING” :

NON-PROBABILIST Specimen-taking

The paper bag
contains nothing
but a **VALUELESS**
DANGEROUS
SPECIMEN !

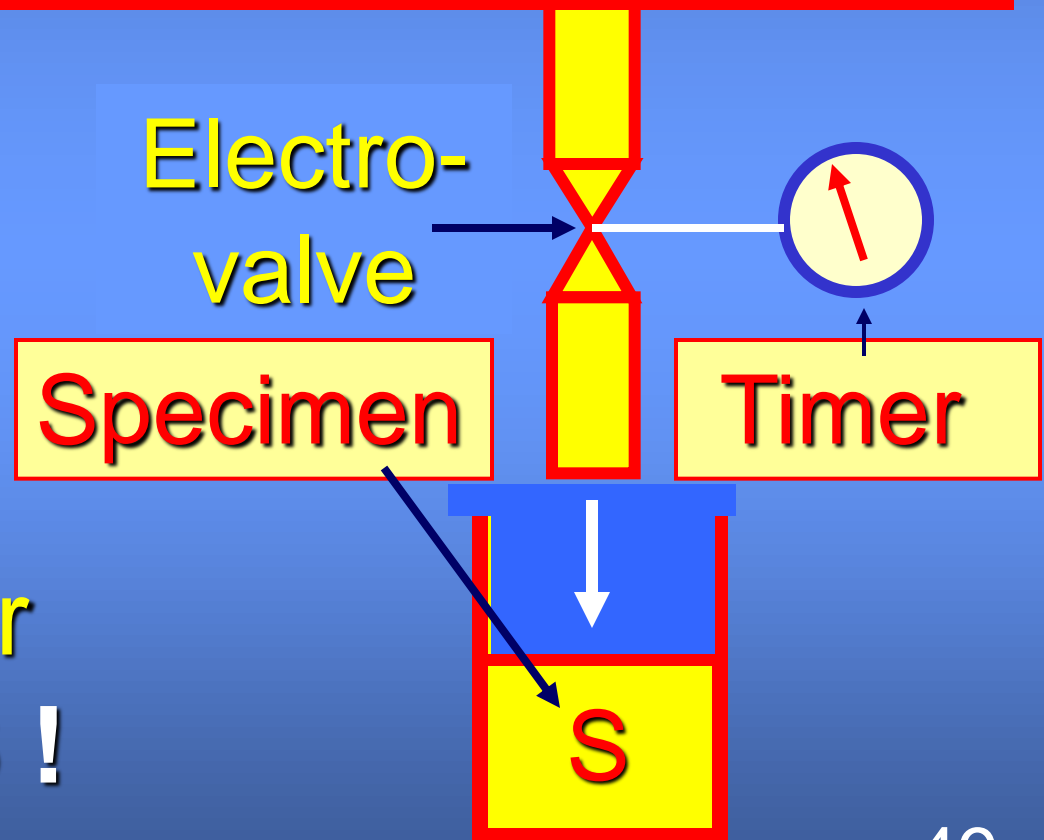


To conditioning and marketing

Example borrowed from chemical industries

LOT L TO BE SAMPLED

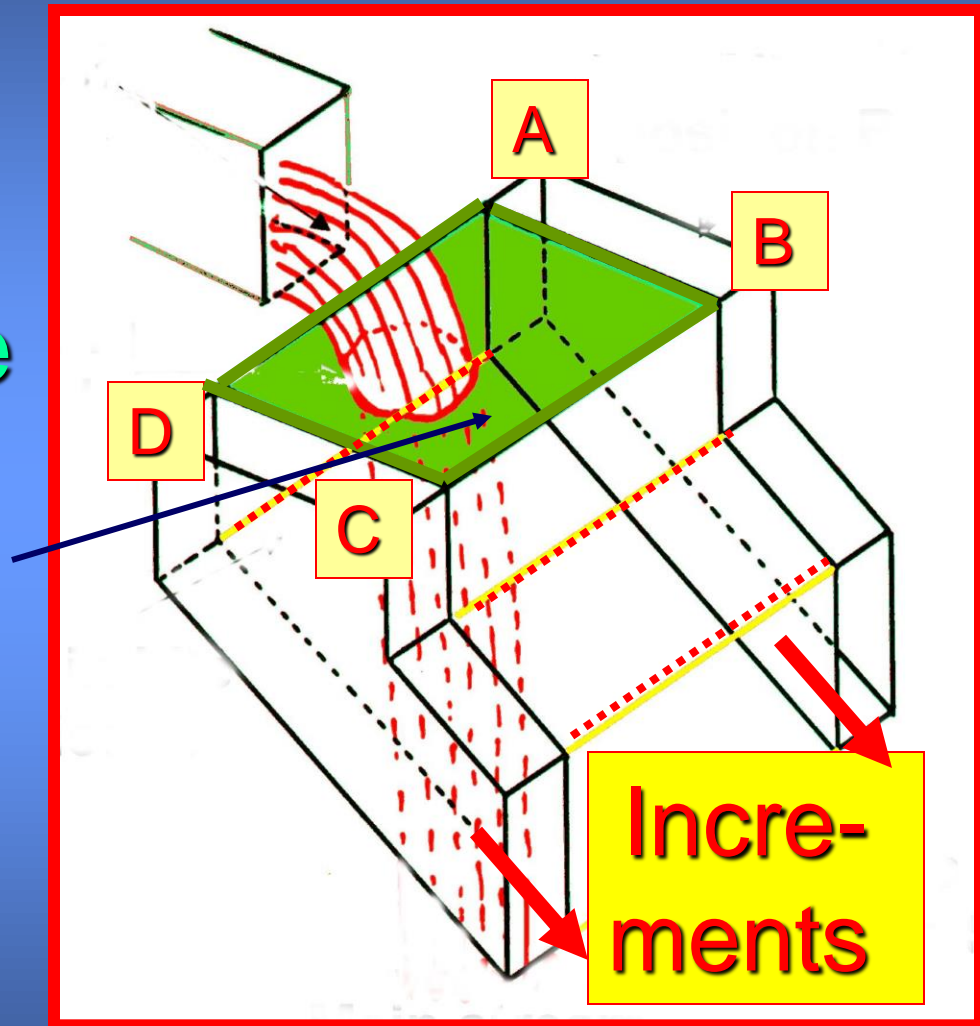
Hypothesis
of homogeneity
throughout the
cross-section.
**Practically never
observed. BIAS !**



THE SAMPLING PLANE ABCD

Straight-path
cutter.

Sampling plane
≡ Area scanned
by the cutter
(green area
ABCD)

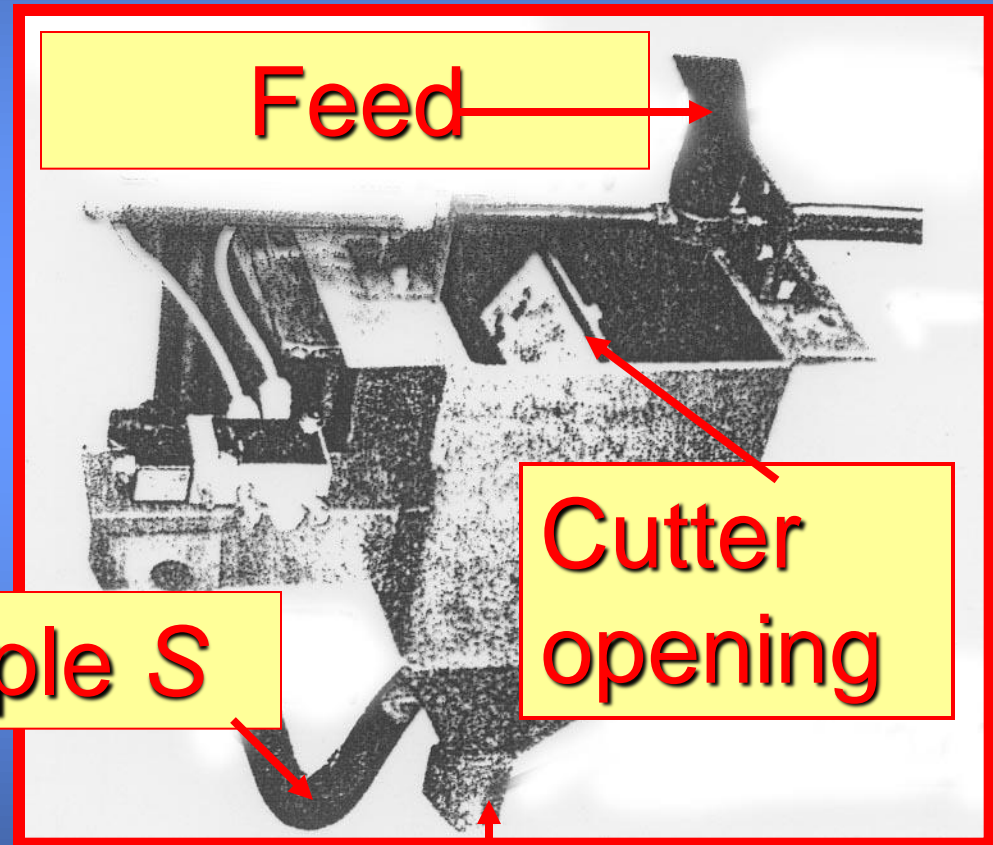


STRAIGHT-PATH STREAM SAMPLER

Very small
flow-rate (few t/h)

Hydraulic drive

Velocity
often non-
uniform.



Main stream to process

REVOLVING SAMPLER (Vezin)

Small to very small flow-rates

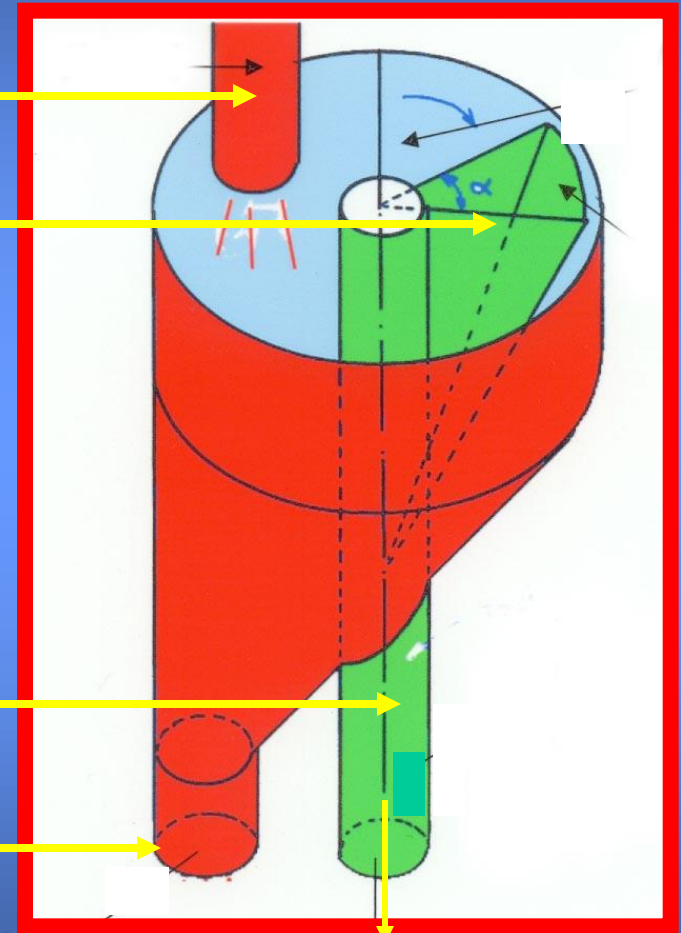
Stream to be sampled

Increment cutter
opening

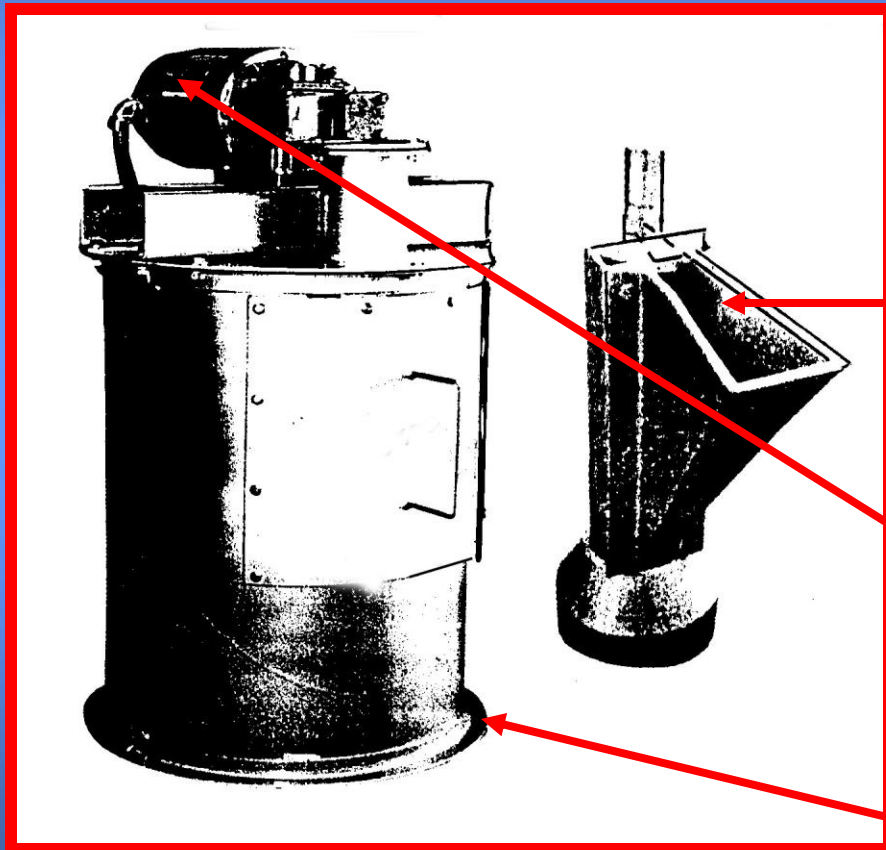
Axial pipe to drive cutter
and evacuate increments

Main stream to process

Increments and sample



REVOLVING SAMPLER (Vezin-type)



Usually correct

Cutter and
axial tube
to evacuate
increments

Electric drive
(correct)

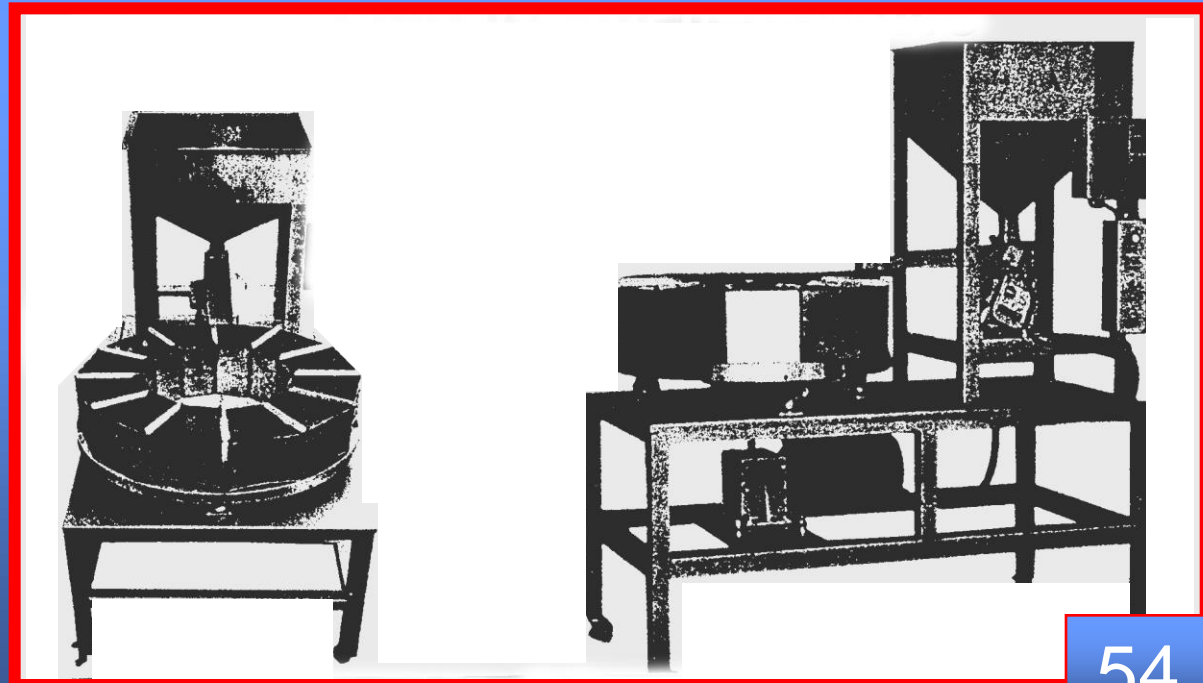
External view

REVOLVING SPLITTER WITH A FIXED FEEDER

Turn-table with **N** removable containers passing through the discharge of a vibrating feeder. Lot **L** is split between **N** twin-samples.

Very small
capacity

Commercial
operations.

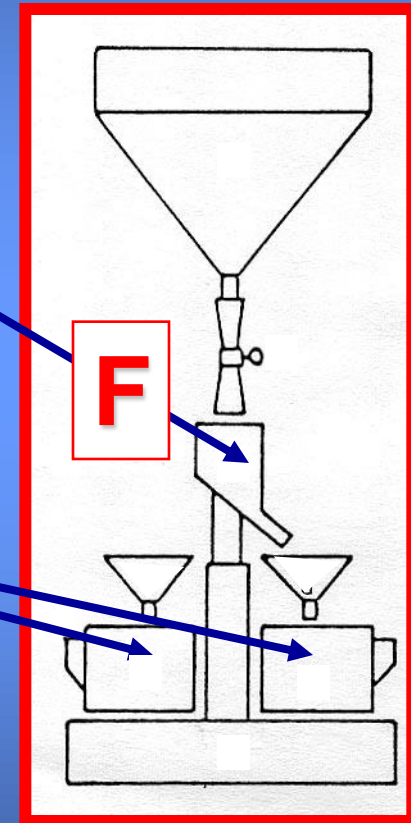


REVOLVING FEEDER SECTORIAL DIVIDER

Usually correct **laboratory**
but also pilot plant device. The
feeder **F** revolves about its axis.

Irrespective of its mass, lot **L**
can be divided into **N** twin-
fractions. Each of these can be
taken as a $1/N$ th sample of **L**.

Typically : $N = 6, 8, 12$



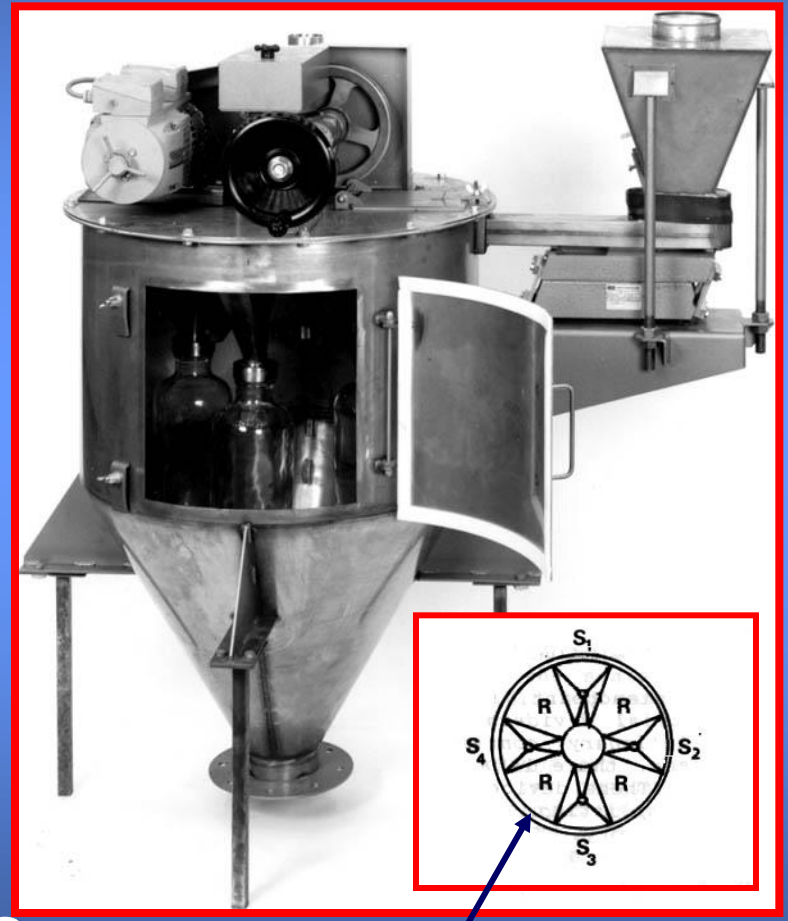
DIRECT BOTTLING UP REVOLVING SPLITTER

MINEMET device.

Fixed feeder.

**Lot L is distributed
between 4 two-liter
jars + a reject R.**

Completely enclosed
dust-proof device desi-
gned to split very fine
and dry uranate powders.



Revolving Cutter

DIRECT BOTTLING UP REVOLVING SPLITTER

Same design, smaller scale.

MINEMET device.

Final splitting of
uranate powders
(Equatorial Africa)

Air moisture and
dust perfectly
controlled. Feeds 12×250 ml jars contain-
ing final samples (to parties / reserve). 57

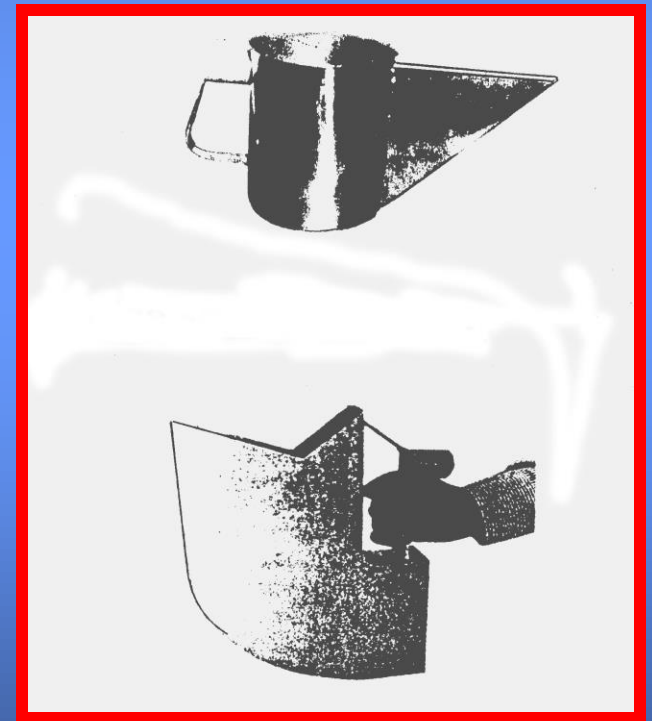


HAND SAMPLING OF FLOWING STREAMS

Incorrect and DANGEROUS instruments

Some of these, have been standardized (!) and are still all too often used at industrial scale.

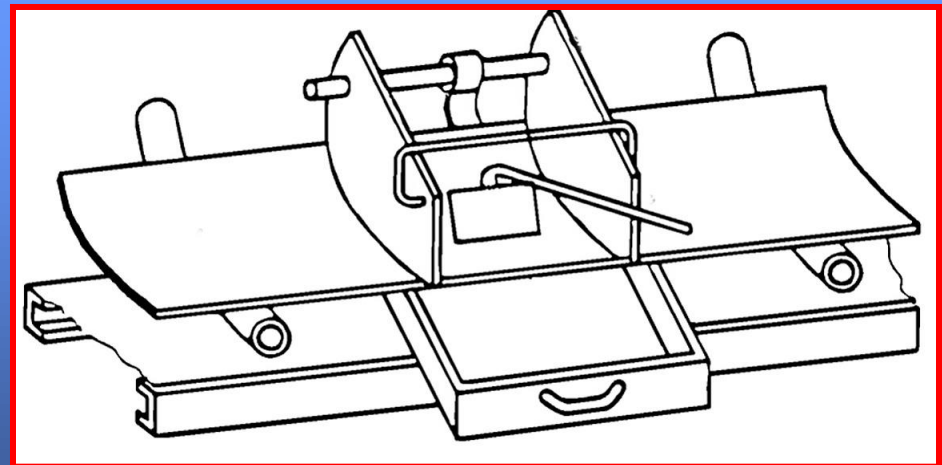
To be avoided whatever the flow-rate.



STOPPED BELT SAMPLING OF FLOWING STREAMS OF SOLIDS

This hand method is especially implemented as a “**REFERENCE METHOD**” when testing a sampler for bias. **Never implemented industrially.**

Correct when the operator does not try to cheat !



OVERFLOW « SAMPLER »

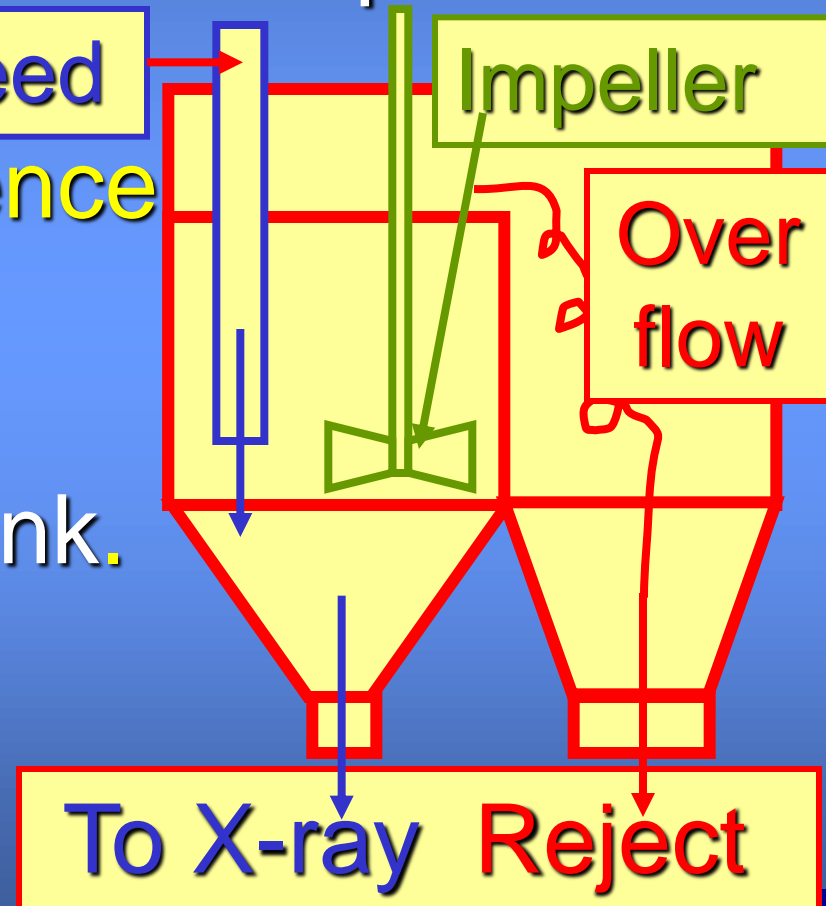
ENSURING UNIFORM FLOW-RATE

One of the most incorrect samplers !

The problem was to **Feed** an X-ray fluorescence analyser with a uniform flow-rate of pulp.

Hence the overflow tank.

HUGE BIASES
due to differential gravity segregation



HOME-MADE ILL-DESIGNED SPECIMEN-TAKER

Nickel mine in South-America

Geological drilling ♦ Biased ♦ Unreliable



QUALITATIVE APPROACH

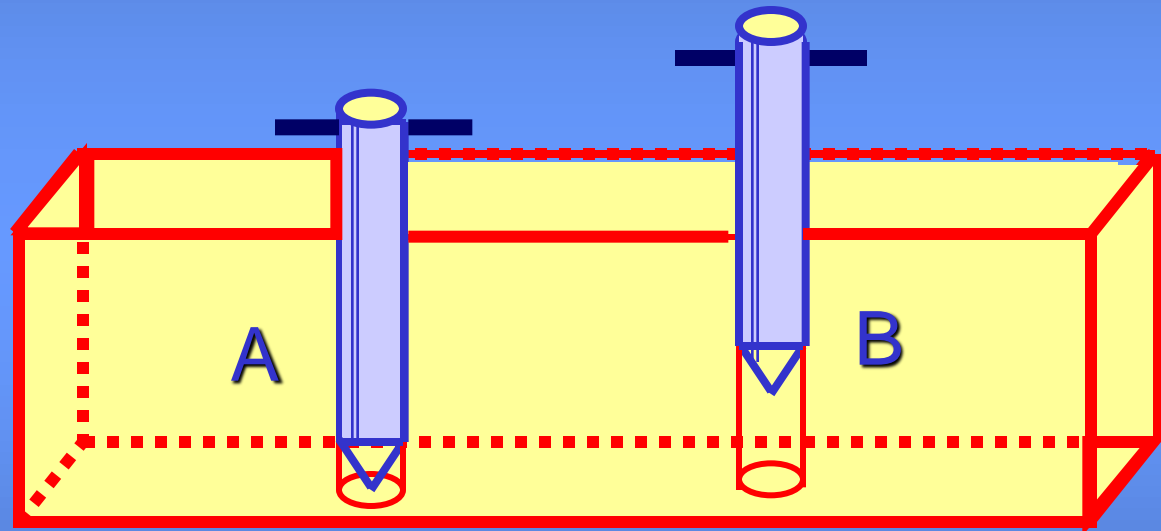
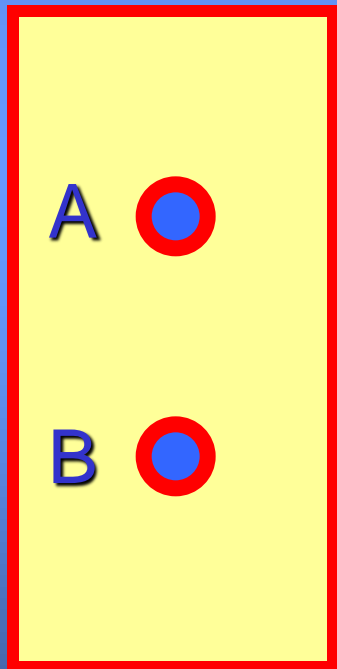
THE “SAMPLING” OF TWO-DIMENSIONAL MATERIAL BATCHES

Actually “Non-Probabilist Specimen-taking” 62

TWO-DIMENSIONAL SAMPLING ♦

HAND-PROBE ♦

EXAMPLE : Truckload of loose sandy material.



Model sample : whole cylinder
Specimen obtained : **BIASED**

Practically, never goes to bottom : Ex. 63

TWO-DIMENSIONAL SAMPLING

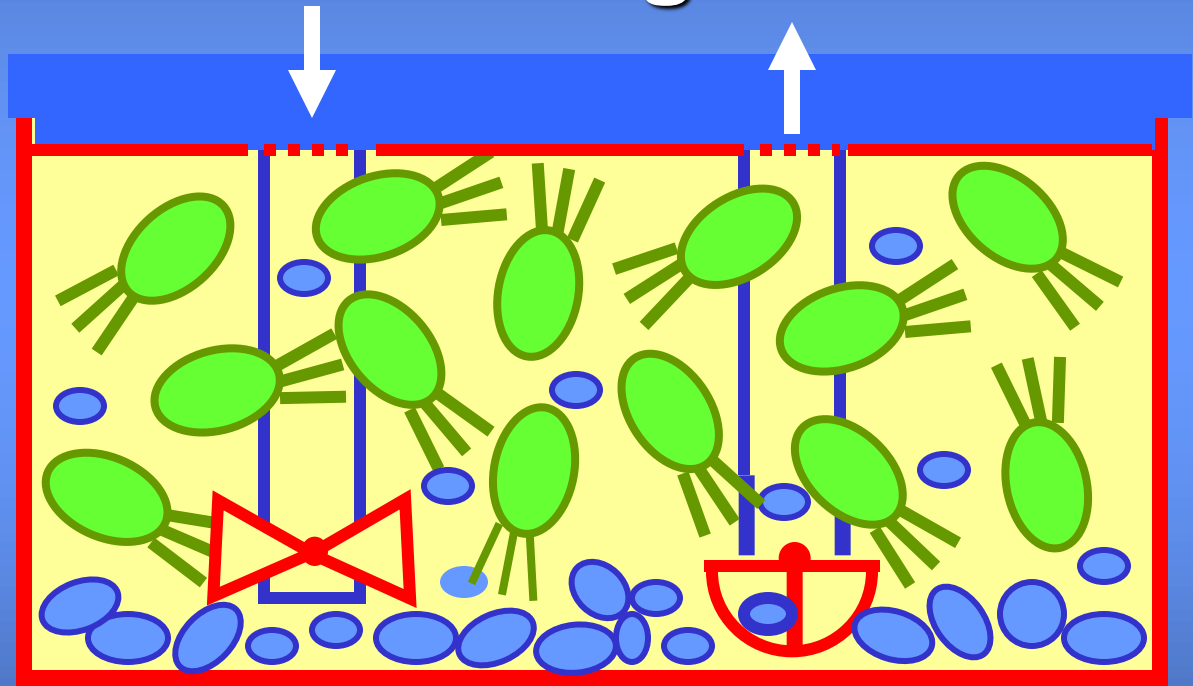
MECHANICAL PROBE

EXAMPLE : Truckload of sugar beets.

Hydraulically
driven probe

Flint pebbles
deposited on
truck bottom
by producers

prior to loading. 1000 % bias on $\text{SiO}_2\%$



DELIBERATE DEFRAUDING

QUALITATIVE APPROACH

THE “SAMPLING” OF THREE-DIMENSIONAL MATERIAL BATCHES

Actually “Non-Probabilist Specimen-taking” 65

« SPECIMEN-TAKING » BY VISUAL SELECTION

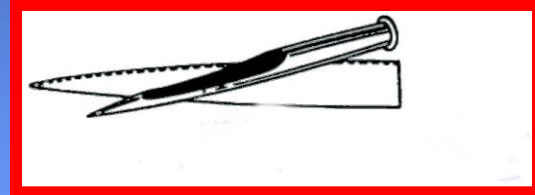
The operator, a “Sworn Sampler”, chooses the fragments he regards as “representative” of the lot.

Actually unreliable “Specimen taking”.



“SPECIMEN-TAKING” BY MEANS OF THIEVES, PROBES, AUGERS

THIEF : spear-like implement.
Textile sacks of coffee, etc.

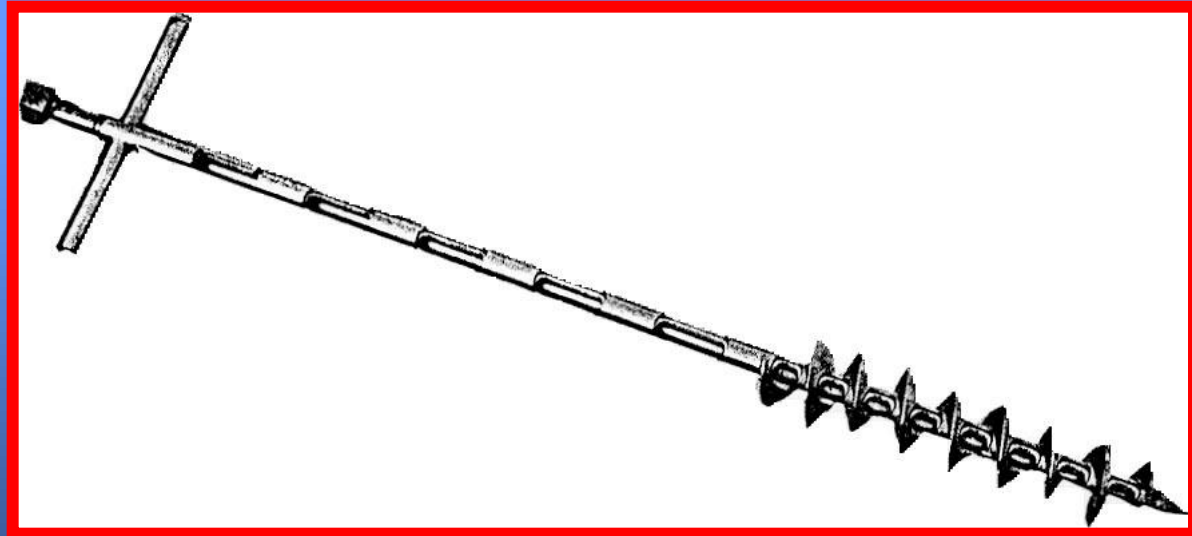


PROBE :



AUGER =

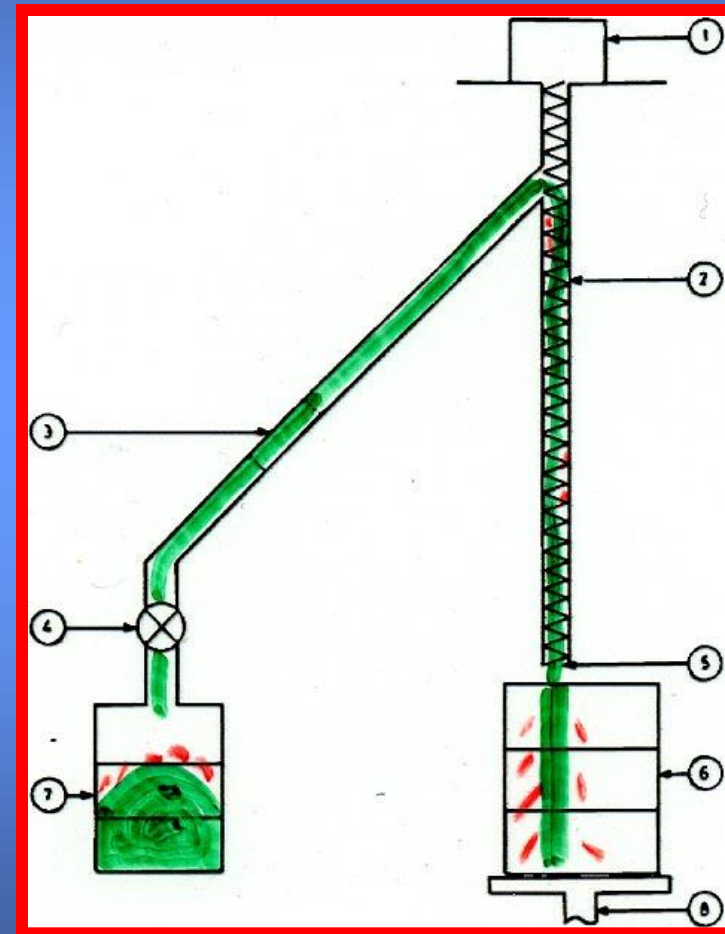
Corkscrew
probe



“SAMPLING” DRUMS BY MEANS OF A SHEATHED AUGER

Experiment carried out by the U.S. Atomic Energy Commission (1960s) on uranates. Found biased. The drum can be emptied !

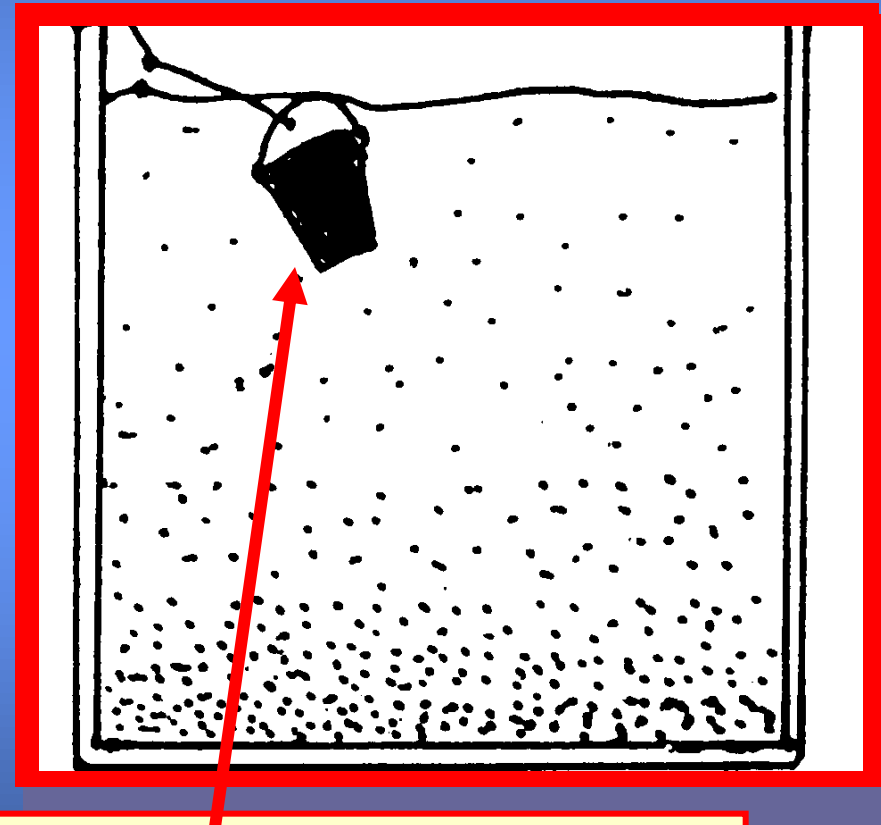
CROSS-STREAM
SAMPLING WAS
PREFERRED AND
ADOPTED AS A ROUTINE METHOD



“SAMPLING” A TANKFUL OF A GROUND SOLID « PULP »

CHEAP INCORRECT MANUAL METHOD !

Due to selective
gravity segregation,
the “specimens” are
always biased.



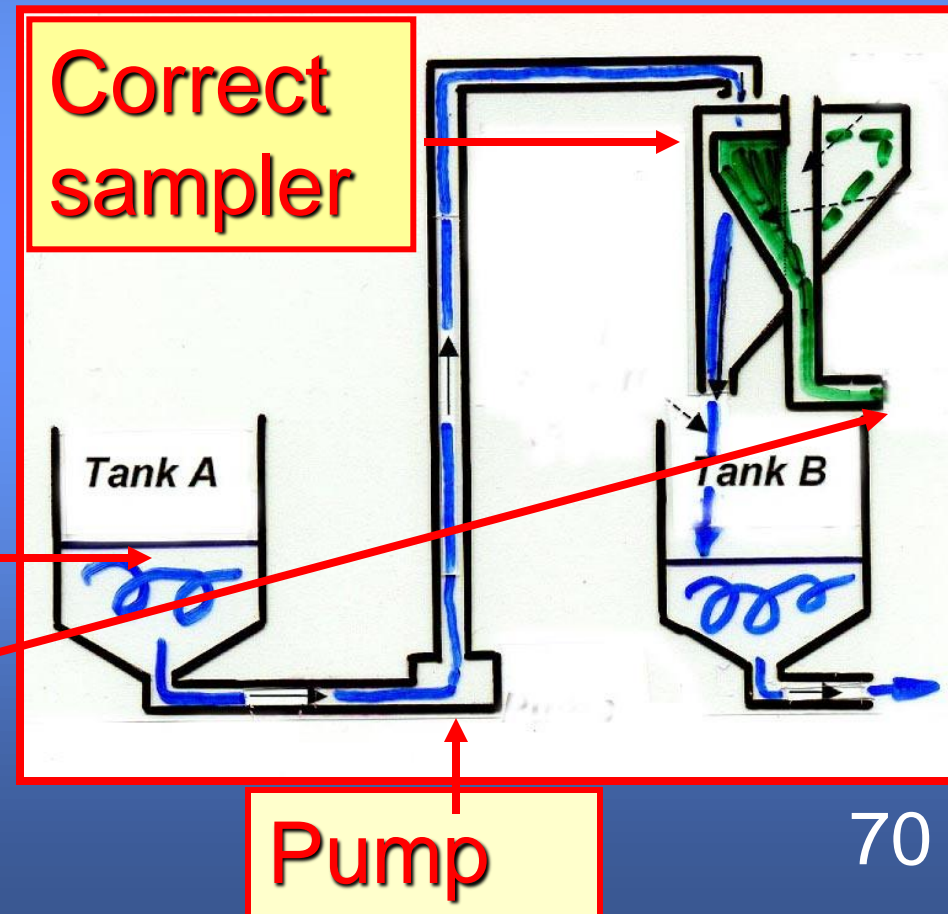
Bucket = specimen

« CORRECT SAMPLING » OF A TANKFUL

ONLY CORRECT METHOD

Lot L is transformed
into a one-dimensional
object and sam-
pled as such.

Lot L
Sample S



« SAMPLING » A GOLD-CYANIDE SOLUTION FOR ITS ANALYSIS

Assayed for Au by Atomic Absorbption :

The solution was ...

CLEAR : absence of solids / gels

LIMPID : no optical distortion

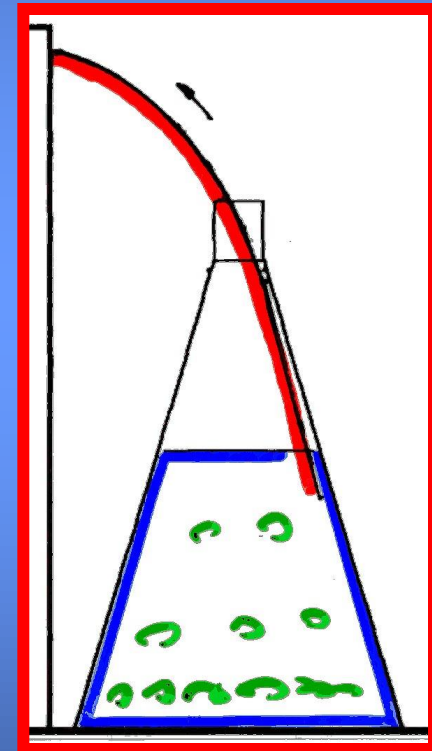
After 11 hours of free settling :

1st reading (no stirring) : 387

2nd reading (mild stirring) : 550

3 to 8 (violent shaking) : 850 ± 2

TRAP !



INVISIBLE GOLD IONS DO SEGREGATE

QUALITATIVE APPROACH

NON-PROBABILIST
SAMPLING \equiv
SPECIMEN-TAKING

Examples and Dangers

NON-PROBABILIST SPECIMEN-TAKING

Part of the lot is an unknown quantity

**THEREFORE there can be NO THEORY
of NON-PROBABILIST SAMPLING**

**NON-PROBABILIST SAMPLING ERRORS
ARE THEREFORE UNPREDICTABLE**

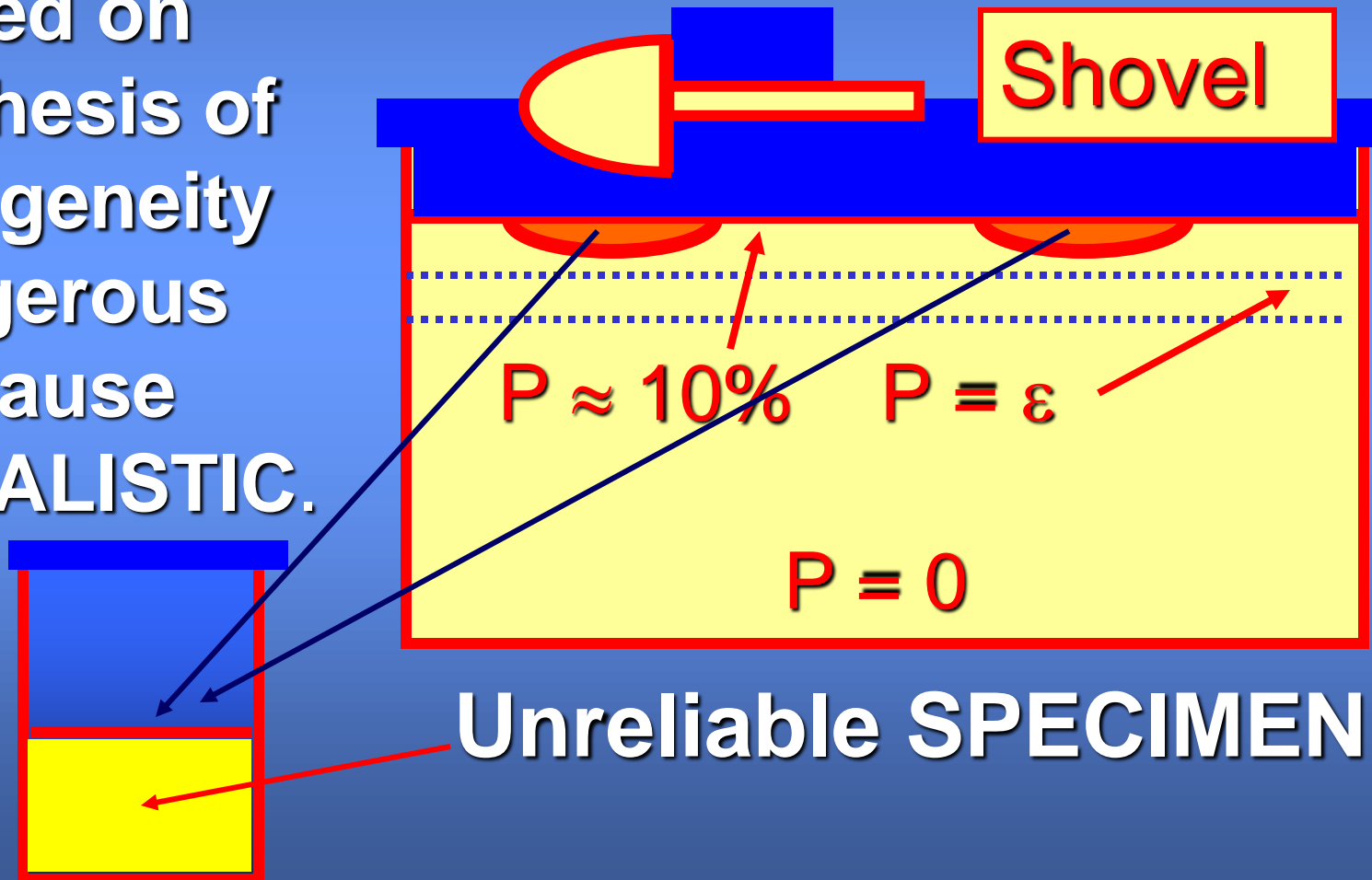
**EXPERIENCE SHOWS they are
USUALLY UNACCEPTABLE**

- **THE THEORY PRESENTED HERE IS A THEORY OF PROBABILIST SAMPLING CORRECT OR INCORRECT**
- **NON-PROBABILIST SAMPLING PROVIDES NOTHING BUT UNRELIABLE SPECIMENS**
- **THESE SPECIMENS ARE DANGEROUS, ESPECIALLY IN COMMERCIAL SAMPLING**

PICKING = SPECIMEN-TAKING

Picking on top of a container : **drum, bag, waggon, truck, etc.** Most accessible part of lot L.

Based on
hypothesis of
homogeneity
dangerous
because
UNREALISTIC.



EXAMPLES OF COSTLY SPECIMEN-TAKING ERRORS

- MINERAL PROCESSING PLANT : designed on basis of biased specimens: loss $\$10^7$ (1960)
- TRADE OF TIN CONCENTRATE : use of method No.3 providing biased specimens ...



Loss of \$7 million over three years (cheating at the smelter).

- **BLAST HOLE CUTTINGS** : copper-gold mine. Specimen-taking as shown cost the mine \$8 million a year. Two sources of loss :
 - valuable ore was sent to waste pile,
 - waste was sent to plant.

A correct sampling plant was amortized in weeks !

