SEVENTH PART

QUANTITATIVE APPROACH

Answer to question **« How Much ? »**

Second Case : TIME SERIES

ESSENCE OF THE PROBLEM

Lot L flows between instants $t = 0$ and $t = T_L$

The unknown of the problem is the average grade a_L of L. Grade $a(t)$ is the proportion of component A in the slice of matter that crosses the « sampling plane » between time t and $t + dt$. Grade a_L of L is the integral mean of $a(t)$ whose algebraic expression is never known.

$$\mathbf{a_L \equiv \frac{1}{T_L} \int_0^{T_L} a(t)\, dt}$$

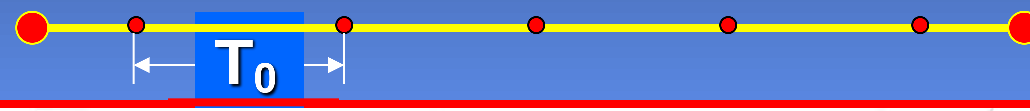
The best we can do is to assay samples taken at a uniform interval in segment $[0, T_L]$

This operation can be broken up into two error-generating steps :

- **SELECTION** of Q extensionless Point-Increments I_q (instant t_q). The segment $[0, T_L]$ is replaced by a series of Q values of $a(t_q)$.
- **MATERIALIZATION** of Points-Increments, i.e. transformation of points I_q into Material-Increments ready for assay ➔ est $[a(t_q)]$.

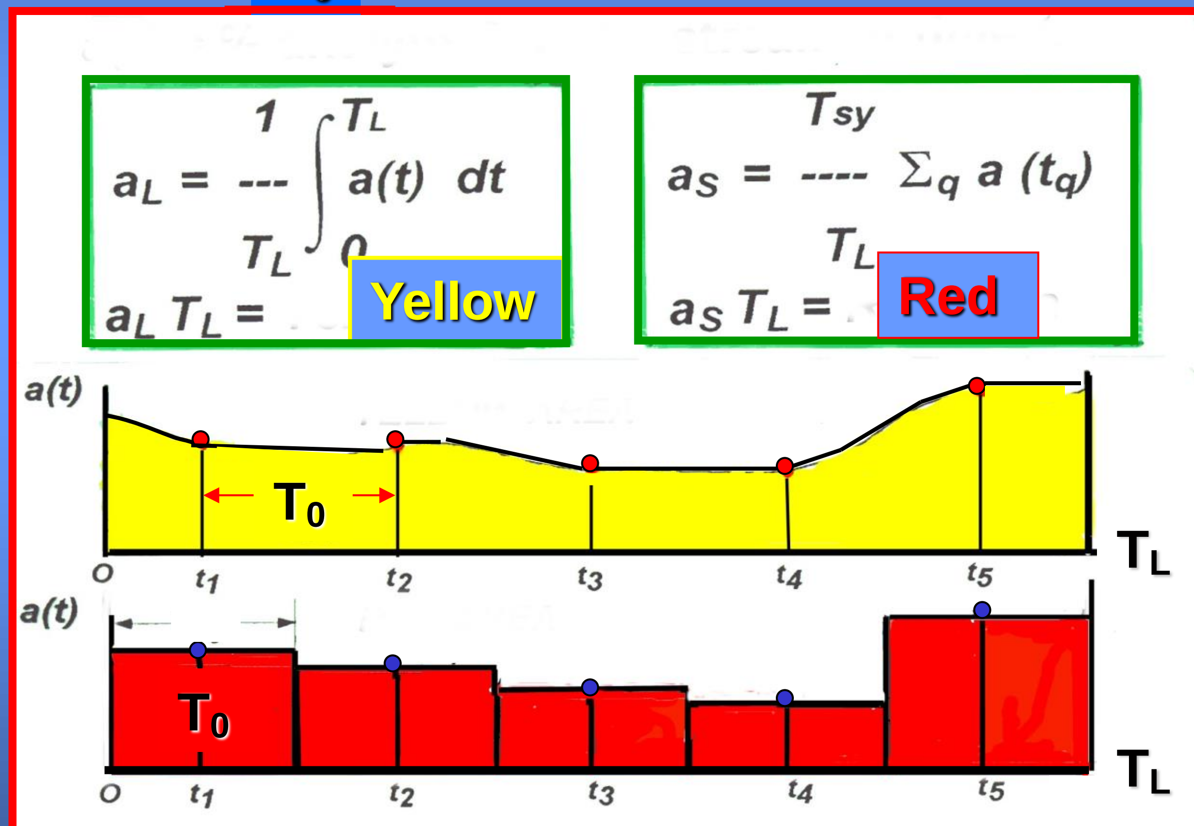
SELECTION OF POINT-INCREMENTS

Time axis : $0 \quad t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \; T_L$



Yellow area is the true value of product $a_L\ T_L$

Red area is the value of product $a_S\ T_L$



Generates Point Selection Error PSE …

POINT SELECTION ERROR PSE

Lot **L** flows between instants **$t = 0$** and **$t = T_L$**. The **Point Selection Error PSE** is the error committed when the segment **$[0, T_L]$** is replaced by a **series of Q Point-Increment I_q**. PSE is studied farther on.

$$\mathbf{PSE} \equiv \frac{a_S T_L - a_L T_L}{a_L T_L} \equiv \frac{\text{Red minus Yellow}}{\text{Yellow}}$$

There remains to materialize the immaterial, extensionless points **I_q** …

MATERIALIZATION OF THE POINT-INCREMENTS

The problem is to transform a series of…

- **IMMATERIAL, EXTENSIONLESS POINTS** on the time axis into …
- **MATERIAL INCREMENTS and then into a MATERIAL SAMPLE** obtained by gathering the increments that collectively represent the lot L .

BREAKING UP THE TOTAL SAMPLING ERROR TSE

To the …

POINT SELECTION ERROR PSE

adds up the …

POINT MATERIALIZATION ERROR PME.

The **TOTAL SAMPLING ERROR TSE** can therefore be expressed as follows :

Total Sampling Error TSE

$$TSE \equiv PSE + PME$$

The materialization of the point-increments can be broken up into a sequence of four logical steps …

- **DELIMITATION**
- **DISCRETIZATION**
- **EXTRACTION**
- **PREPARATION**

that we are going to review now.

- **DELIMITATION** of a volume of matter : this Volume-Increment **VI** does not take the discrete nature of the fragments into account
- **DISCRETIZATION** ≡ definition of a Model Material-Increment **MMI** made of fragments « pertaining to » the Volume-Increment,
- **EXTRACTION** of Actual Material-Increment **AMI** from the Model Material-Increment
- **PREPARATION** ≡ Gathering / Processing of Actual Material-Increments and Sample to form the **Sample Ready for Assay**.

INCREMENT DELIMITATION
♦ THE VOLUME-INCREMENT ♦

Delimitation is a purely geometrical step which consists in transforming an …

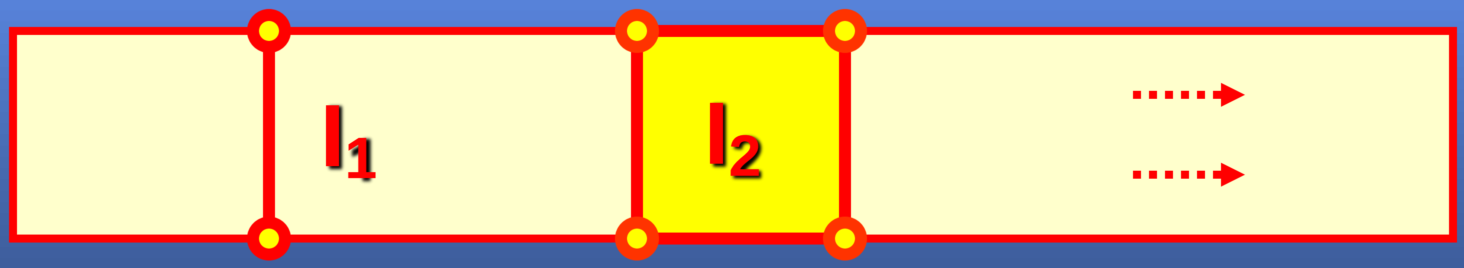
- **IMMATERIAL, EXTENSIONLESS POINT :** the « Point-Increment » into a …
- **VOLUME :** the three-dimensional « Volume-Increment ».

This operation can be broken up into several steps that may be correct or incorrect …

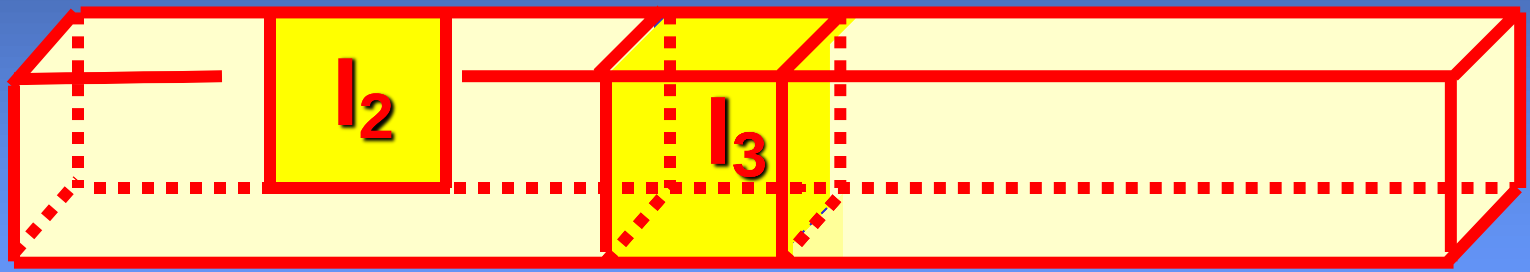
We define a certain volume of matter that does not respect the fragments boundaries. This aspect is taken care of in the second step

- **The Zero-dimensional Point I_0** is extended into a **One-dimensional Segment I_1**

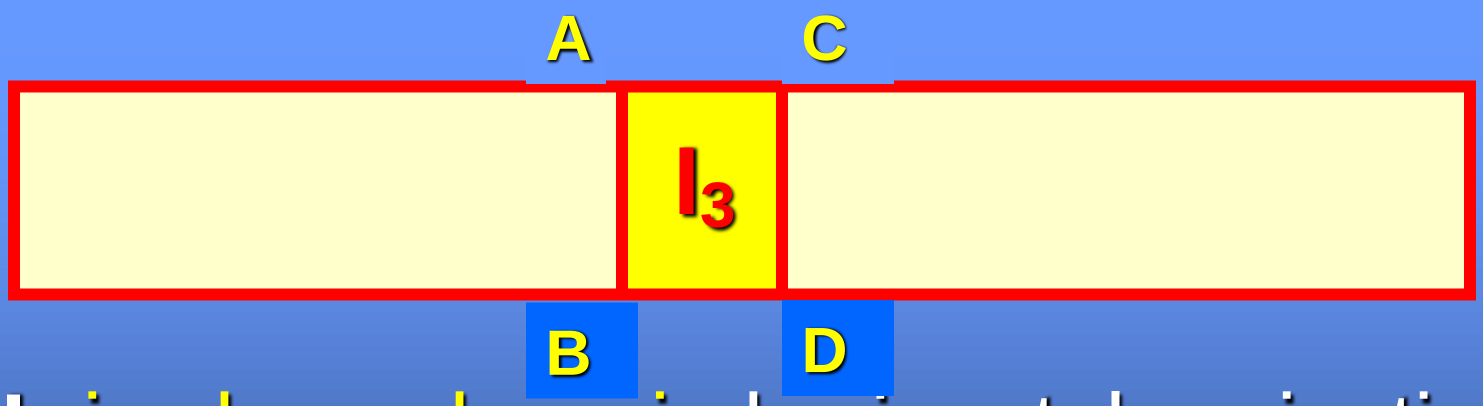
- **The One-dimensional Segment I_1** is extended into a **Two-dimensional Surface I_2**



- **The Two-dimensional Surface I_2** is extended into a **Three-dimensional Volume I_3**



I_3 is called the « Volume-Increment »

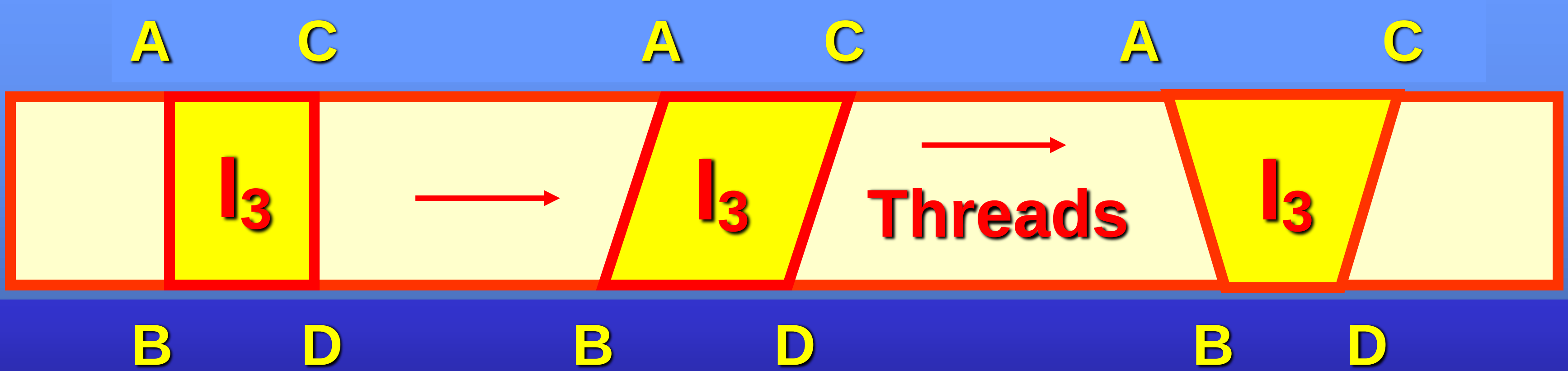


I_3 is shown here in horizontal projection

A B C D on which we will now reason.

CROSS-STREAM ♦ CORRECT INCREMENT DELIMITATION

DEFINITION : **P ≡ constant.** Involves that all threads of the stream be cut during the same lapse of time, which entails that faces **AB** and **CD** be parallel. Examples :



Correct **Correct** **Incorrect**

CORRECT DELIMITATION

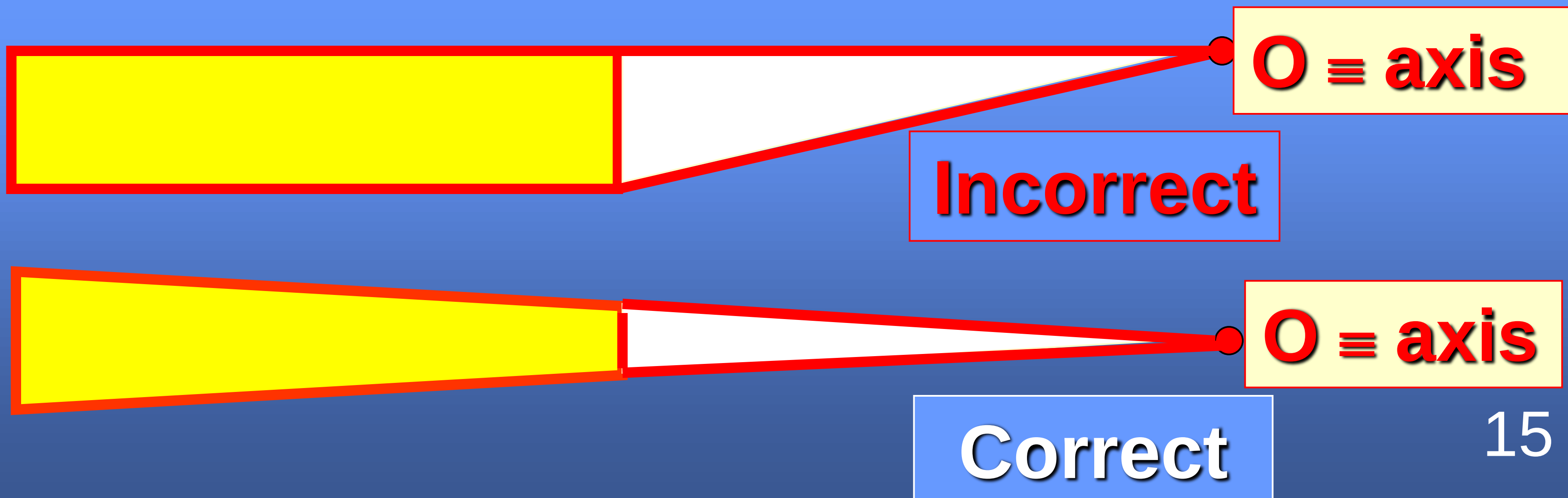
- **CUTTER GEOMETRY :** it is correct when, and only when (three cases) :
  - **Straight path cutter** : edges are parallel,
  - **Circular path cutter** : edges are radial,
  - **Undefined path** (hand sampling) : there is no correct geometry. Never correct.
- **CUTTER VELOCITY :** it is correct when, and only when, the velocity is uniform during the stream crossing.

INCORRECT DELIMITATION

- **INCORRECT CUTTER GEOMETRY :**

  ♦ **Straight-path cutter** : incorrect when deformed by wear or obstructed.

  ♦ **Circular-path device**: ill-designed cutter studied in Angola in 1973 (frequent)



- **CUTTER VELOCITY** is **INCORRECT** when the cutter ...

  - **Does not reach its nominal velocity**. The idle positions are too near the stream.

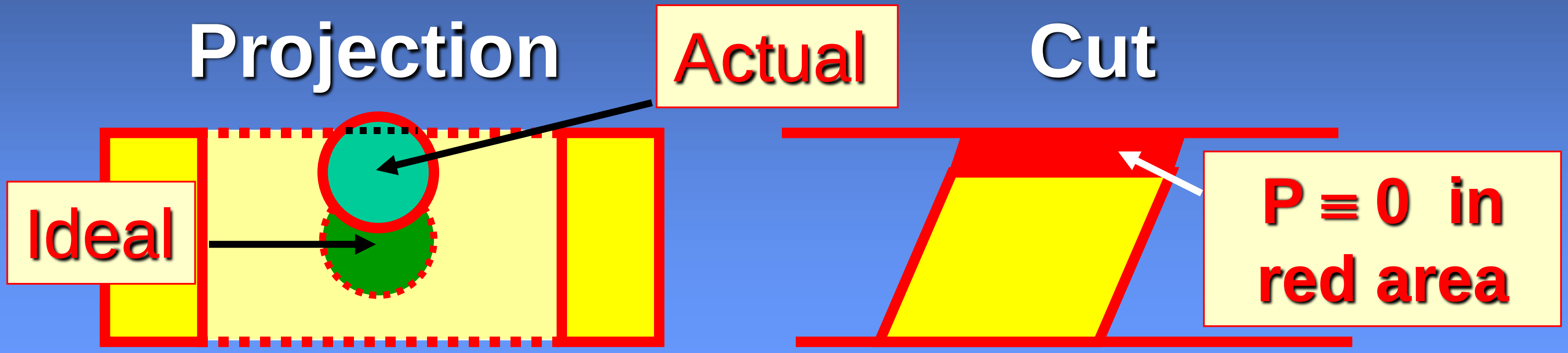
  - **Slows down when cutting the stream**. The drive is not powerful enough.

  - **Hydraulic, pneumatic, magnetic, hand drives**. Cannot warrant a uniform velocity.

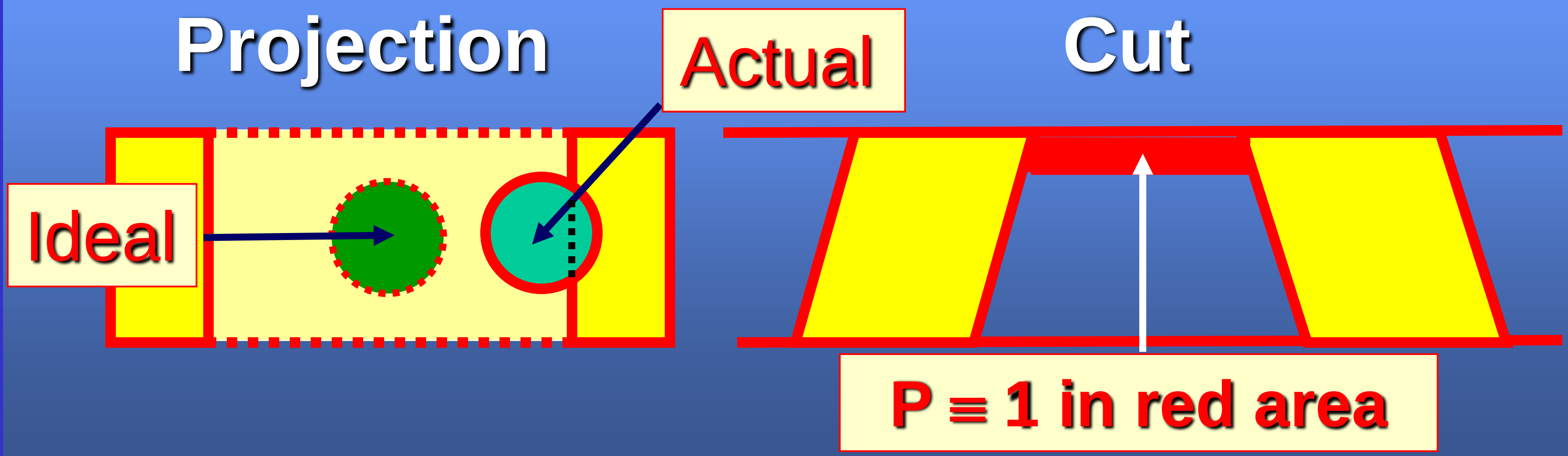
ELECTRIC DRIVE ALONE, when powerful enough, warrants a UNIFORM VELOCITY

• Incorrect lay-out of a correct cutter

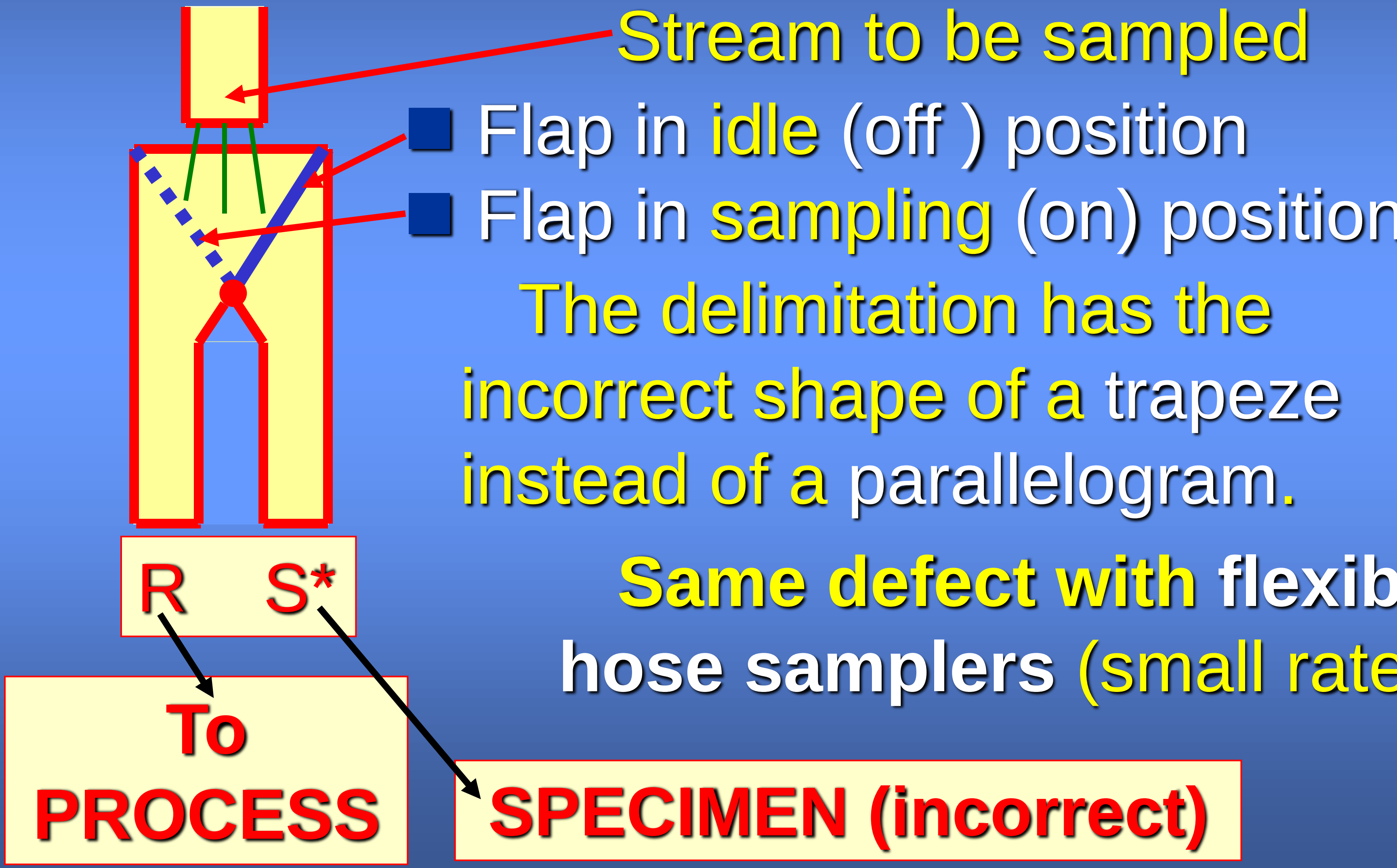
- ♦ Never cuts a certain fraction of the stream



- ♦ Cuts a certain selective fraction of the stream when idle (rebounds, splashes).



- **Structurally incorrect design**
  - **Flap samplers** (usually « home-made »)



The delimitation has the incorrect shape of a trapeze instead of a parallelogram.

Same defect with flexible hose samplers (small rates)

DELIMITATION : Two-dimensional projection

- ♦ **CORRECT**

1. Model : Frame on a stopped belt,
2. Cross-stream from left to right,
3. Cross-stream from right to left.

- ♦ **INCORRECT**

4. Flap and flexible hose,
5. Cross-stream that **slows down**,
6. Cross-stream that does not reach its nominal velocity and **is still accelerating**.

INCREMENT DELIMITATION ERROR IDE

When delimitation does not respect conditions of correctness, an error takes place …

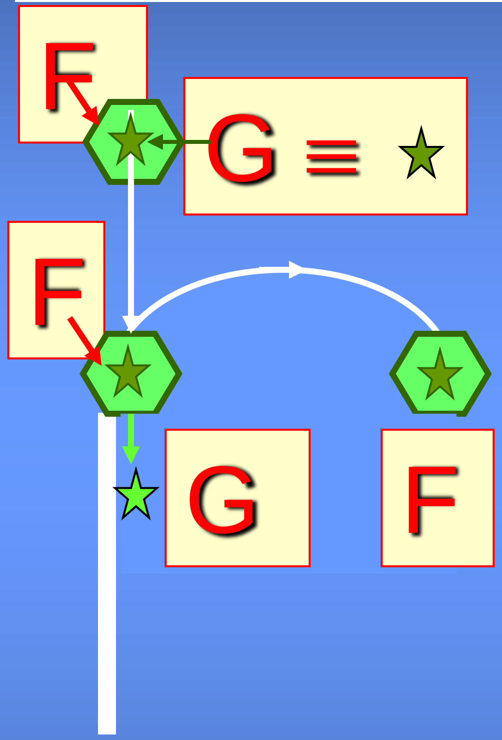
INCREMENT DELIMITATION ERROR IDE

This error cannot be estimated beforehand. Experience shows **it can be very large**.

The only efficient strategy with **IDE** is to …

ELIMINATE IDE BY IMPLEMENTING CORRECT EQUIPMENT CORRECTLY

DISCRETIZATION
♦ MODEL MATERIAL-INCREMENT ♦



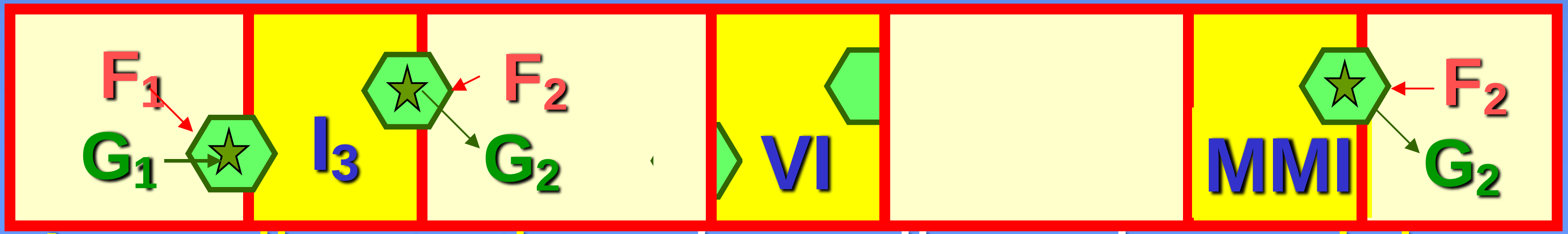
Rebound on cutter edge

THE REBOUNDING RULE

A fragment **F** bounces towards the cutter side that contains its center of gravity **G**. Fragments behave as if they were condensed in **G**. All fragments whose center of gravity falls within the **Volume-Increment** make up the …

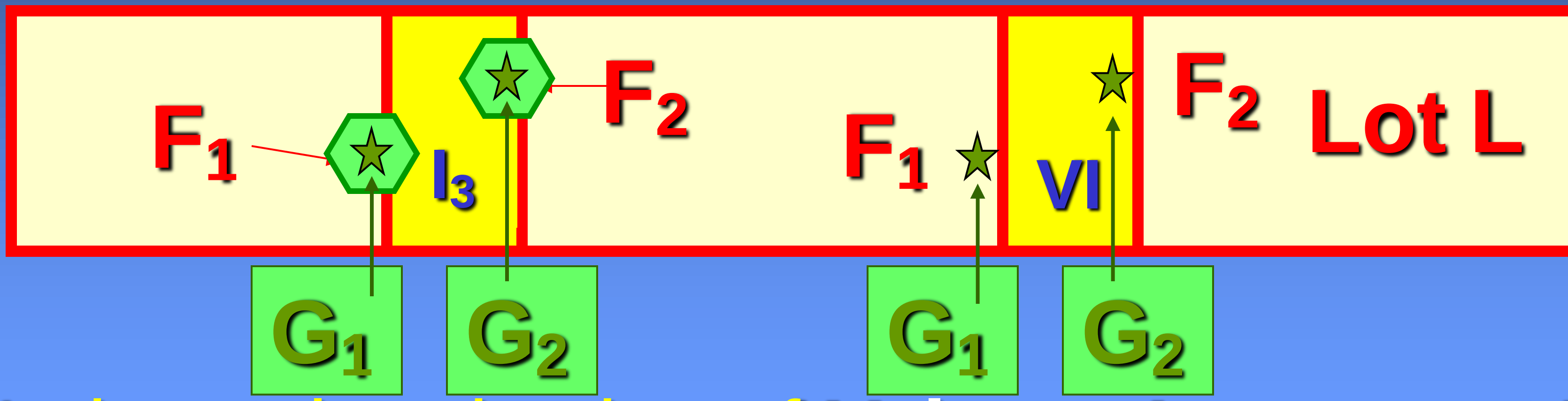
Model Material-Increment

We cannot isolate the matter contained in the **« Volume-Increment VI »** (center : green areas ). We therefore have to transform the volume **I_3** into a group of fragments.



According to the rebounding rule model, each fragment **F** behaves as if it was condensed in **G**. **G_2** falls within the **« Volume-Increment VI »** (center), then **F_2 belongs** to the **« Model Material-Increment MMI »** (right).
F_1 does not.

The « Model Material-Increment » MMI



Horizontal projection of **Volume-Increment VI**

According to the rebounding rule F_1 and F_2 are condensed in ☆ G_1 and ☆ G_2 . Therefore :

G_2 falls within VI ➔ F_2 does belong to MMI

G_1 falls outside VI ➔ F_1 does NOT belong to MMI

Due to the existence of the structural **« Constitutional Heterogeneity »** of the material, the **Model Material-Increment MMI** differs from the **Volume-Increment VI.** This difference is a random variable.

The variance of the population of all possible « Model Material Increments » MMI is nothing other than the variance of the structural …

« Fundamental Sampling Error FSE »

defined by the zero-dimensional sampling model.

INCREMENT EXTRACTION
♦ ACTUAL MATERIAL-INCREMENT ♦

We have abstractedly defined the composition of **« Model Material-Increment MMI »**

There remains to concretely **extract** an **« Actual Material-Increment AMI ».**

This operation may be differential or selective : the extraction probability is no longer uniform. In other words the selection may be incorrect. If that is the case, we observe an

INCREMENT EXTRACTION ERROR IXE

INCREMENT EXTRACTION ERROR IXE

An « **Increment Extraction Error IXE** » takes place as soon as the rebounding rule is not **UNIFORMLY** respected for **ALL** fractions or classes (e.g. size- or density-classes).

This may involve **a selective or differential interaction between the cutter and the material being sampled**.

This is usually due to the inadequacy of the cutter characteristics …

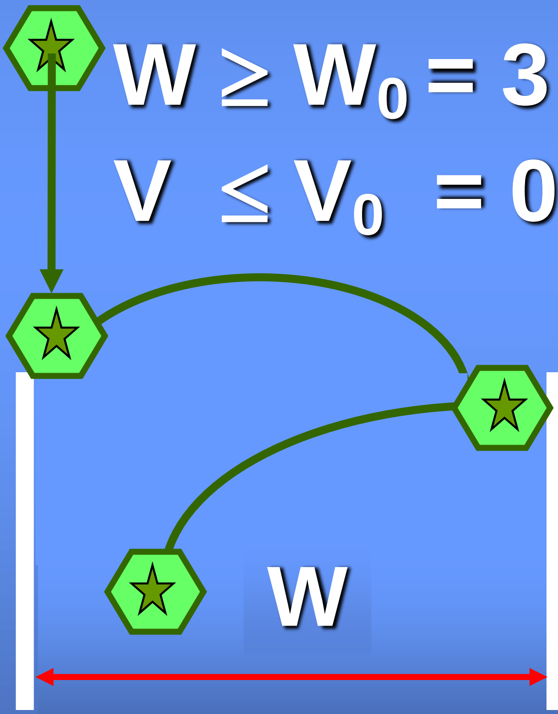
The **two critical parameters** are again :

- the **cutter geometry** : width **W**
- the **cutter velocity V**

Experimental results (1978) show that :

$\mathbf{W \geq W_0 = 3\ d}$ and

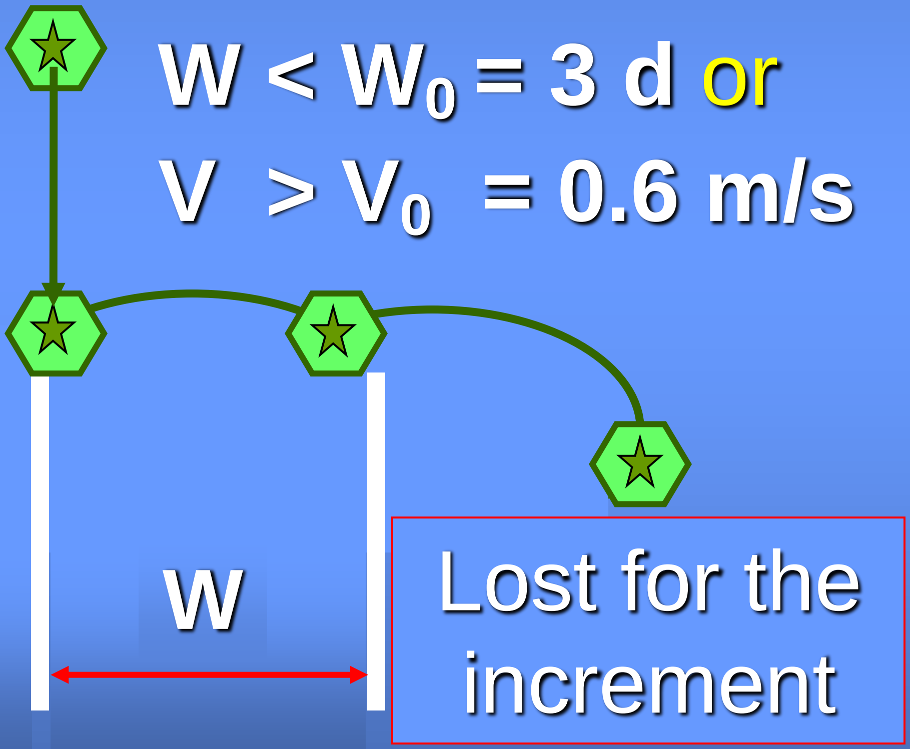
$\mathbf{V \leq V_0 = 0.6\ m/s}$



Correct Extraction

$\mathbf{W < W_0 = 3\ d}$ or

$\mathbf{V > V_0 = 0.6\ m/s}$



Incorrect Extraction

INCREMENT PREPARATION ♦ SAMPLE READY FOR ASSAY ♦

The increments are gathered, transferred, crushed, ground, dried, etc… to form the « **Sample Ready for Assay SRA** ». These operations are potentially error-generating. Correctness demands that increments and sample remain unaltered. If they are altered, an …

INCREMENT or SAMPLE PREPARATION ERROR IPE

takes place …

INCREMENT AND SAMPLE PREPARATION ERROR IPE

We disclosed **six components** to **IPE** …

- **Losses** : all elements entering the cutter must be recovered in sample (e.g. dust …)
- **Contamination** : no extraneous element may be allowed into sample (dust, material belonging to other sample, rust, etc.)
- **Alteration of the chemical composition** e.g. loss of constitutional water upon drying (overdrying of silicates) ...

- **Alteration of the physical composition.**

Specific to moisture and size analysis,

- **UNINTENTIONAL operational mista-**

kes : negligence, handling, labelling, mixing up fractions belonging to different samples ...

- **DELIBERATE TAMPERING WITH SAMPLES OR ASSAY RESULTS**

Defrauding is not unfrequent in trade and environmental control. Examples with **GOLD** (e.g.Borneo : Bre-X !) and **URANIUM**.

CONSEQUENCES OF A CORRECT POINT MATERIALIZATION

- **correct Delimitation** ➔ **IDE ≡ 0**
- **correct Extraction** ➔ **IXE ≡ 0**
- **correct Preparation** ➔ **IPE ≡ 0**

Correct Materialization of Point-Increments

PME ≡ ISE ≡ IDE + IXE + IPE ≡ 0

which entails :

Total Sampling Error TSE

TSE ≡ PSE + CSE

BREAKING UP THE GLOBAL ESTIMATION ERROR GEE

ON THE SCALE OF THE … OVERALL ESTIMATION PROCESS

- **GLOBAL ESTIMATION ERROR GEE**

$$GEE \equiv TSE_1 + TSE_2 + TAE$$

- **TOTAL (PRIMARY) SAMPLING ERROR TSE_1**
- **TOTAL (SECOND.) SAMPLING ERROR TSE_2**
- **TOTAL ANALYTICAL ERROR TAE**

Each sampling stage is a sequence of several sampling sub-stages …

■ ON THE SCALE OF A GIVEN SAMPLING STAGE or SUB-STAGE

such as **PRIMARY** or **SECONDARY** stages

- **TOTAL SAMPLING ERROR TSE**

$$TSE \equiv CSE + ISE$$

- ♦ **CORRECT SAMPLING ERROR CSE**
- ♦ **INCORRECT SAMPLING ERROR ISE**

We have to distinguish between two cases

…

➔ ZERO-DIMENSIONAL MODEL

- TOTAL SAMPLING ERROR **TSE**

$$TSE \equiv CSE + ISE$$

- ♦ CORRECT SAMPLING ERROR **CSE**

$$CSE \equiv FSE + GSE$$

  - ▶ FUNDAMENTAL SAMPLING ERROR **FSE**
  - ▶ GROUPING / SEGREGATION ERROR **GSE**

- ♦ INCORRECT SAMPLING ERROR **ISE**

➔ ONE-DIMENSIONAL MODEL

- TOTAL SAMPLING ERROR TSE

$$TSE \equiv CSE + ISE$$

- ♦ CORRECT SAMPLING ERROR CSE

$$CSE \equiv PSE + PME$$

- ▶ POINT SELECTION ERROR PSE
- ▶ POINT MATERIALIZATION ERROR PME

$$PME \equiv CSE + ISE \equiv (FSE + GSE) + ISE$$

- ☐ FSE and GSE ≡ STRUCTURAL errors
- ☐ ISE ≡ CIRCUMSTANCIAL errors

♦ INCORRECT SAMPLING ERROR ISE

$$ISE \equiv IDE + IXE + IPE$$

- ▶ INCORRECT DELIMITATION ERROR IDE
- ▶ INCORRECT EXTRACTION ERROR IXE
- ▶ INCORRECT PREPARATION ERROR IPE

AT EVERY SAMPLING STAGE or SUB-STAGE

THE TOTAL SAMPLING ERROR TSE IS

$$TSE \equiv CSE + ISE$$

$$TSE \equiv (PSE + FSE + GSE) + \ldots$$

$$\ldots + (IDE + IXE + IPE)$$

VARIANCE OF THE POINT SELECTION ERROR PSE

Point-Selection is usually CORRECT, therefore UNBIASED.

The estimation of the variance requires a new mathematical tool : the **Variogram** introduced by Matheron to quantify the autocorrelation of **space series** of data, we implement it to quantify the autocorrelation of **time series** of data and estimate the variance **σ^2(PSE).**

A lot L flows between instants $t = 0$ and $t = T_L$. The « grade » $a(t)$ is the grade of the slice of matter that flows between instants t and $t + dt$. The unknown a_L is …

$$\boxed{a_L \equiv \frac{1}{T_L} \int_0^{T_L} a(t)\, dt}$$

but the algebraic expression of $a(t)$ is never known. To approach $a(t)$, the best we can do is to extract Q increments I_q at a uniform interval T_0 …

Q increments I_q are extracted from the stream at a uniform interval T_0. There remains …

♦ to weigh and assay them : mass M_q , grade a_q

♦ to compute the heterogeneity h_q of I_q and [defined below] the variogram $v(jT_0)$ of h_q which characterizes the autocorrelation of h(t). To estimate the variance σ^2(PSE), we also have to define the auxiliary and error-generating functions.

DEFINITION OF THE « MODEL VARIOGRAM » of $h(t_q)$

The lot L, grade a_L, is represented by a series of Q point-increments I_q taken at instants $t_q \equiv t_1 + (q - 1)\,T_0$ with $0 < t_1 \leq T_0$.

The **model variogram** involves the **true, unknown values** of $a(t_q)$, $M(t_q)$, $h(t_q)$ …

$$h(t_q) \equiv \frac{a(t_q) - a_L}{a_L} \times \frac{M(t_q)}{M^*(t_q)}$$

Where $M^*(t_q)$ is the average of $M(t_q)$

The **« model variogram »** of the heterogeneity $h_q \equiv h(t_q)$ is defined as follows …

$\Delta h(q, j) \equiv h_{(q+j)} - h_q$: is the « increase » of h_q between t_q and $t_{(q+j)} \equiv t_q + (j - 1)T_0$

The variogram $v(jT_0)$ or, more simply $v(j)$, is the half-mean-square of $\Delta h(q,j)$ (NOT the half-variance as mean $m(\Delta h) \neq 0$).

$$\boxed{v(j) \equiv \frac{\Sigma\, [\Delta h(q, j)]^2}{2\,(Q - j)}}$$

DEFINITION OF THE « EXPERIMENTAL VARIOGRAM »

Practically, we know only experimental estimates of $a(t_q)$, $M(t_q)$ and $h(t_q)$, namely $a_{ex}(t_q)$, $M_{ex}(t_q)$ and $h_{ex}(t_q)$, with :

$$h_{ex}(t_q) \equiv \frac{a_{ex}(t_q) - a_{L\,ex}}{a_{L\,ex}} \times \frac{M_{ex}(t_q)}{M_{ex}^*(t_q)}$$

Where $M_{ex}^*(t_q)$ is the average of $M_{ex}(t_q)$ and $a_{L\,ex}$ the estimate of a_L.

EXPERIMENTAL VARIOGRAM

Let …

v_{ex} (j) : be the « experimental variogram » computed by means of the experimental data a_{ex} (t_q), M_{ex} (t_q) and h_{ex} (t_q),

$\sigma_{ex}{}^2 \equiv v_0$ = a (± constant) variance resulting from the estimation errors of a(t_q), M(t_q) and h(t_q), including the sampling errors.

Theory shows that …

$$\mathbf{v_{ex}\ (j) = v\ (j) + \sigma_{ex}{}^2 = v\ (j) + v_0}$$

The values of the experimental variogram are equal to those of the model variogram increased by a constant variance.

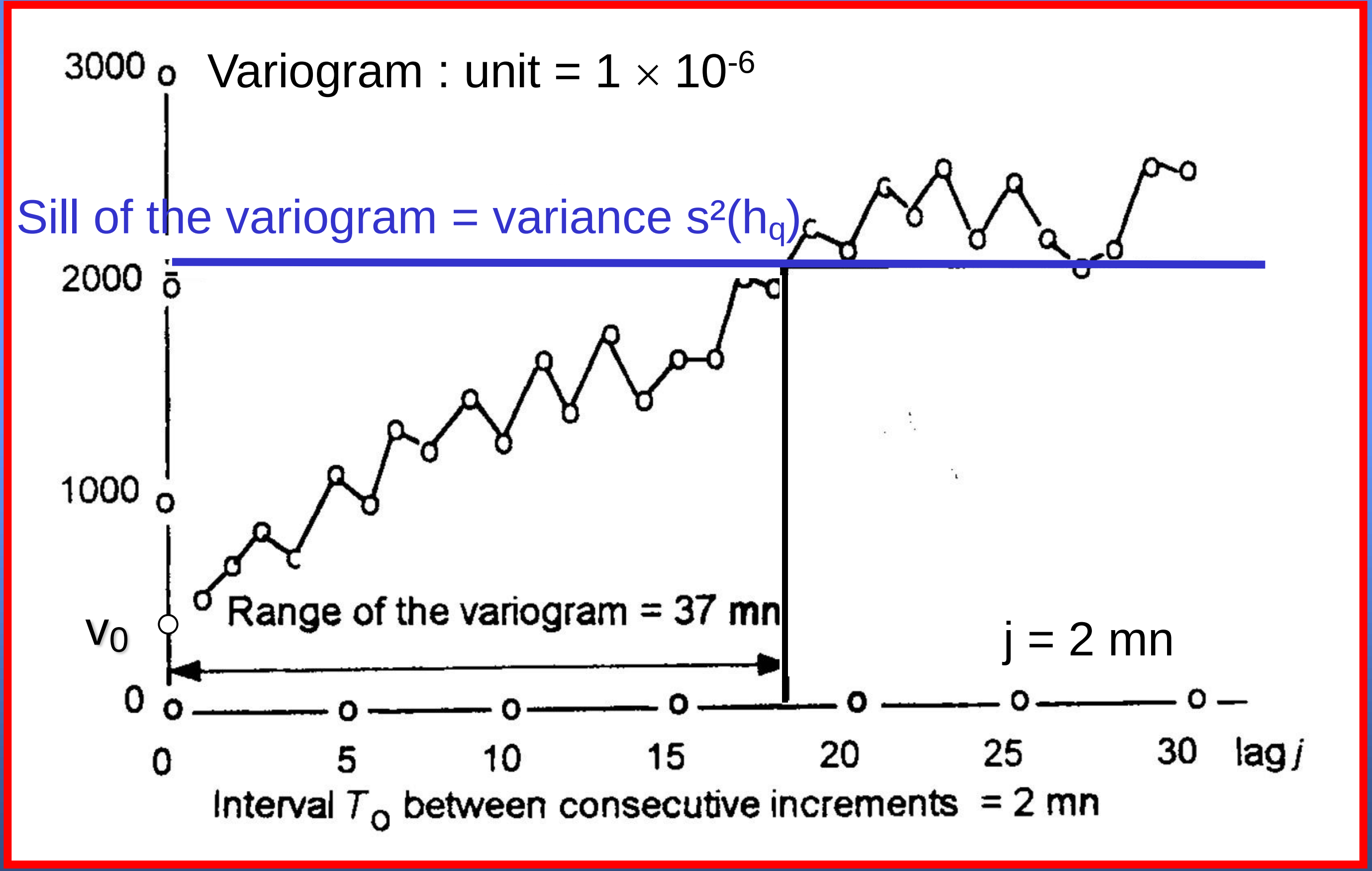
Graphically, the variogram is lifted by the same quantity.

As the **model** variogram is a **CONTINUOUS FUNCTION**, easy to show that $\mathbf{v(0) \equiv 0}$. The value of $\mathbf{v_{ex}(0)}$, intercept of the experimental variogram, is therefore equal to σ_{ex}^2. Hence the practical importance of this intercept.

$$\mathbf{v(0) = 0 \quad \Rightarrow \quad v_{ex}(0) = \sigma_{ex}^2 = v_0 = \text{Intercept}}$$

EXAMPLES OF VARIOGRAMS

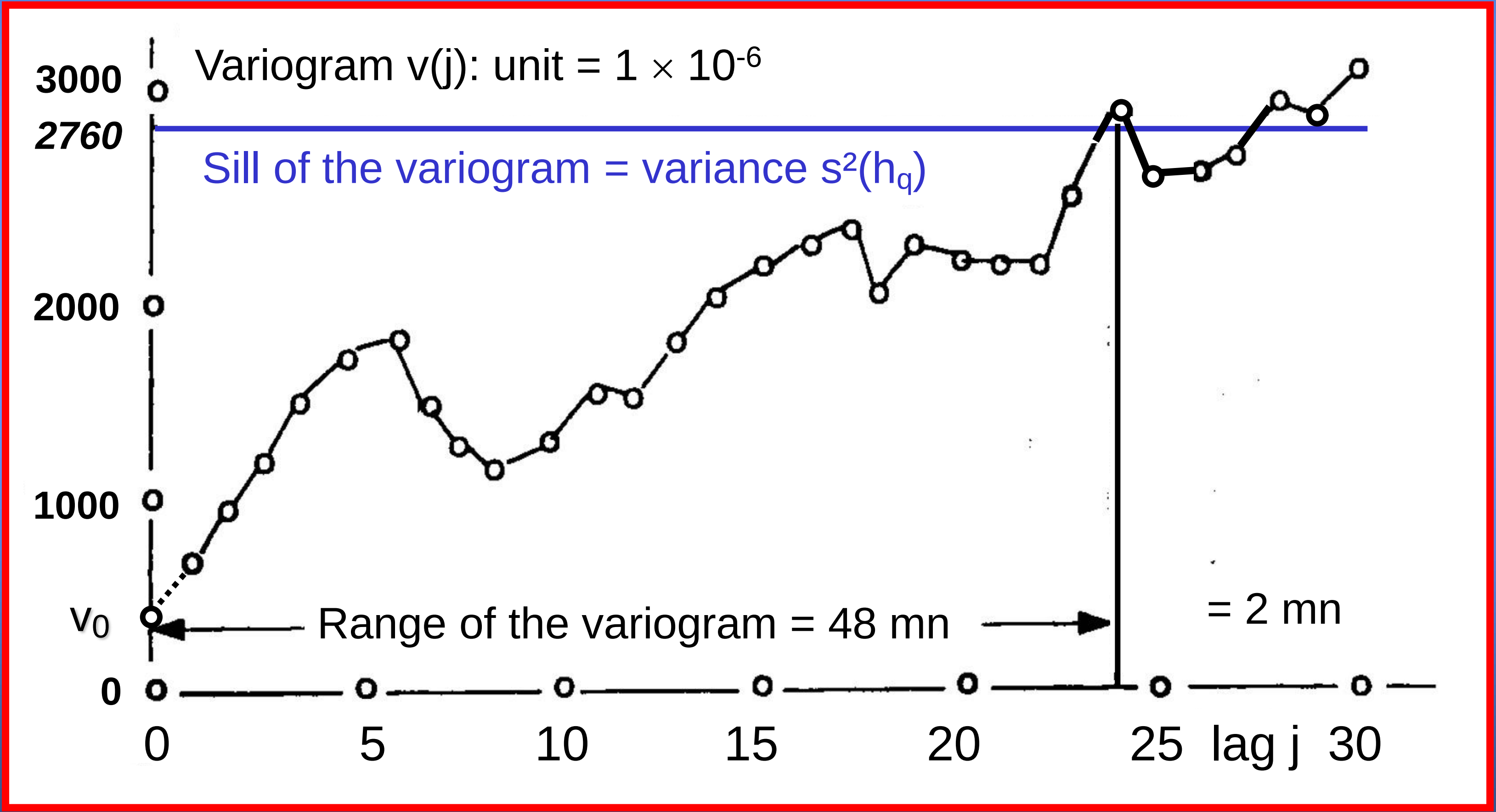
First Example : Feed to a uranium mineral processing plant. Increasing variogram.



EXAMPLES OF VARIOGRAMS

Second Example : Feed to a cement kiln.

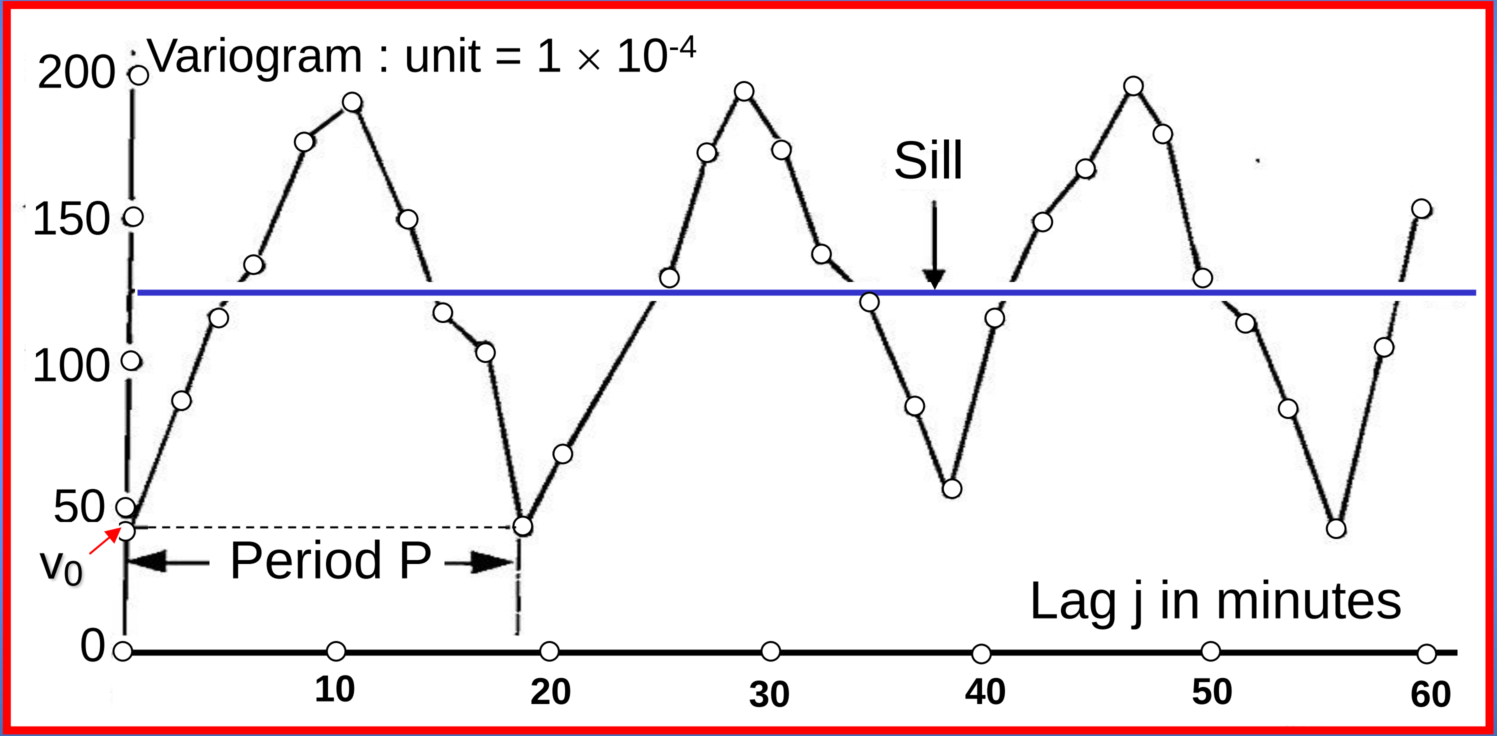
Variogram for heterogeneity of CaO %



EXAMPLES OF VARIOGRAMS

Third Example : Feed to a cement kiln.

Variogram of the increment masses.

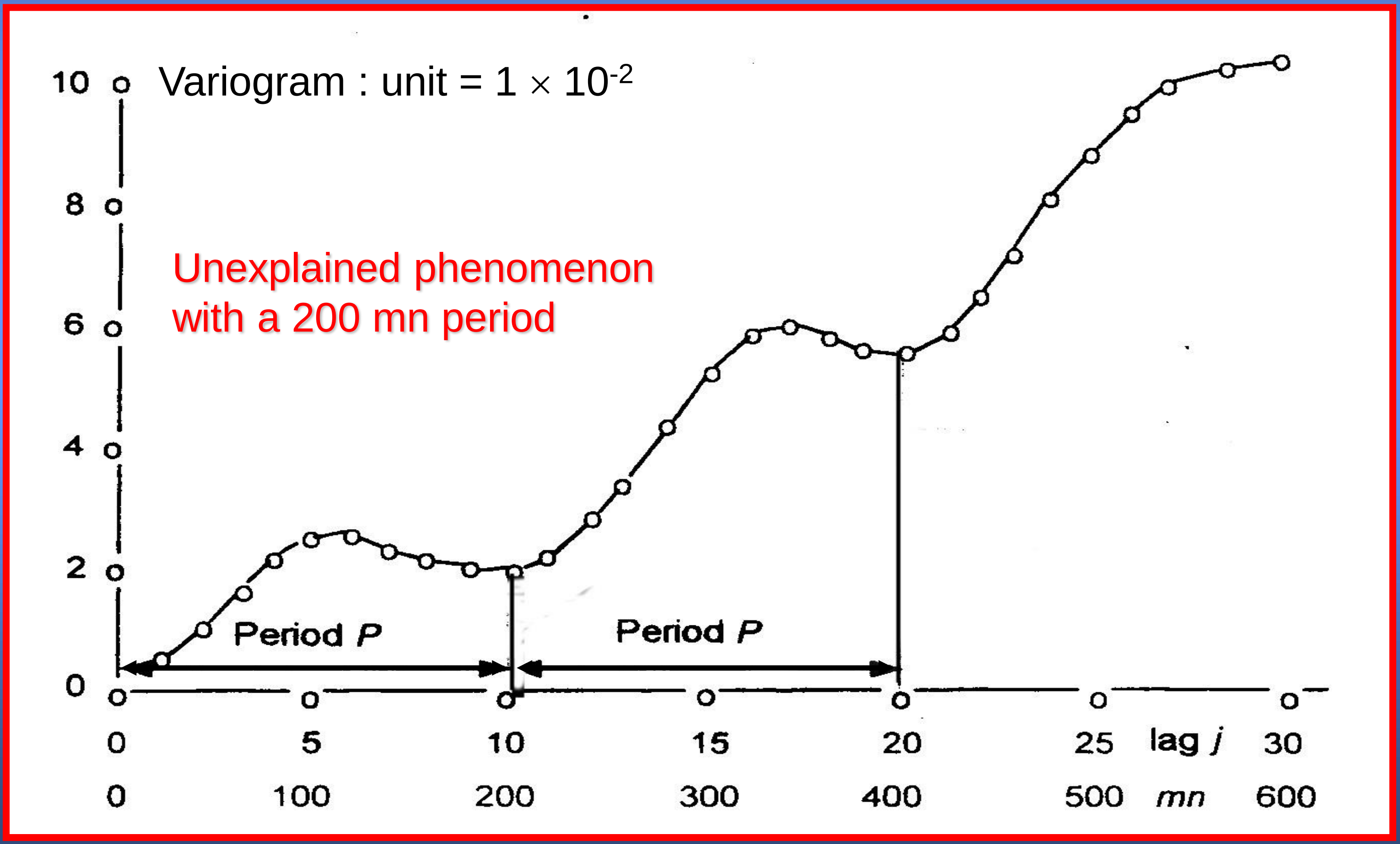


Typical example of a cyclic variogram.

The best estimate of v_0 is the first minimum

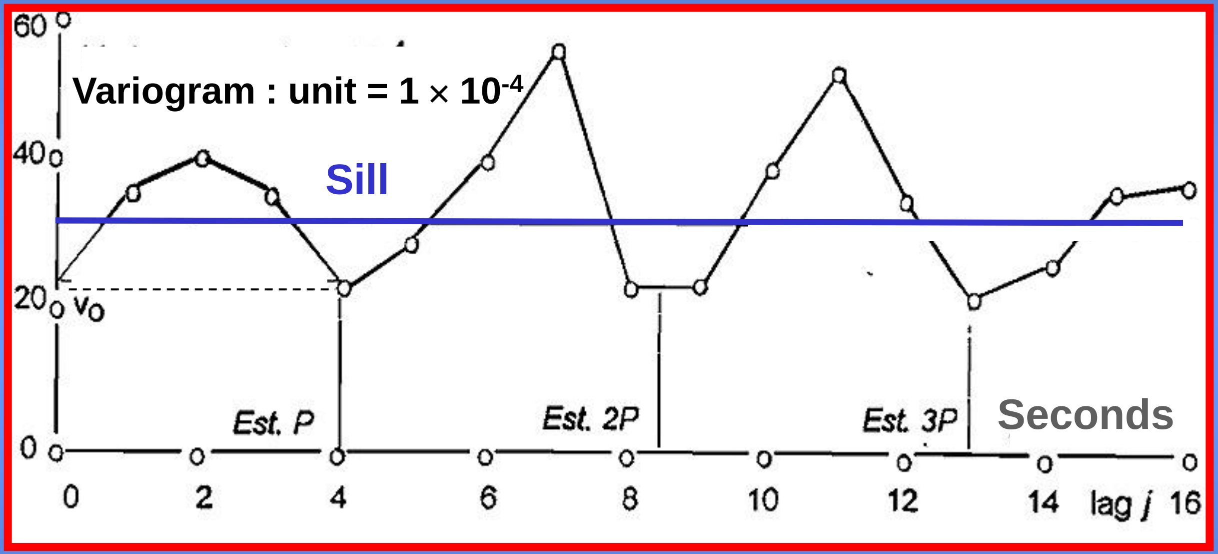
EXAMPLES OF VARIOGRAMS

Fourth Example : Feed to a zinc flotation plant. Variogram for Zn. Cyclic. Long period.



EXAMPLES OF VARIOGRAMS

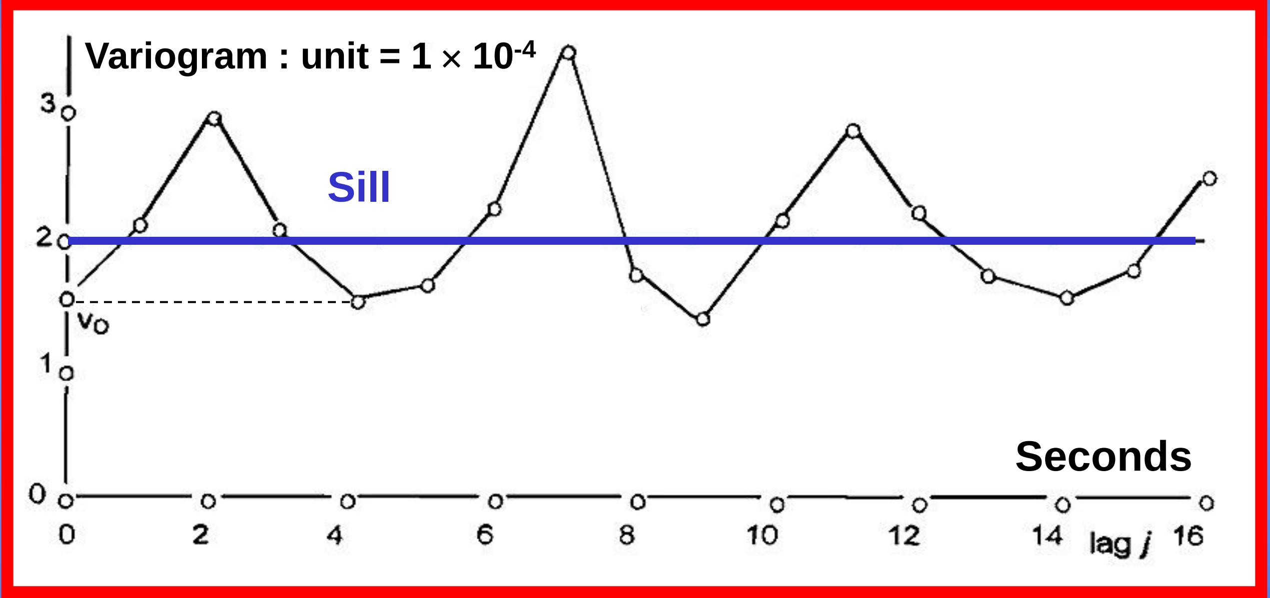
Fifth Example : Output of a bed-blending, fed to a cement kiln. Variogram of SiO_2 %



Cyclic. Very short period (4 to 5 seconds).
The best estimate of v_0 is the first minimum.

EXAMPLES OF VARIOGRAMS

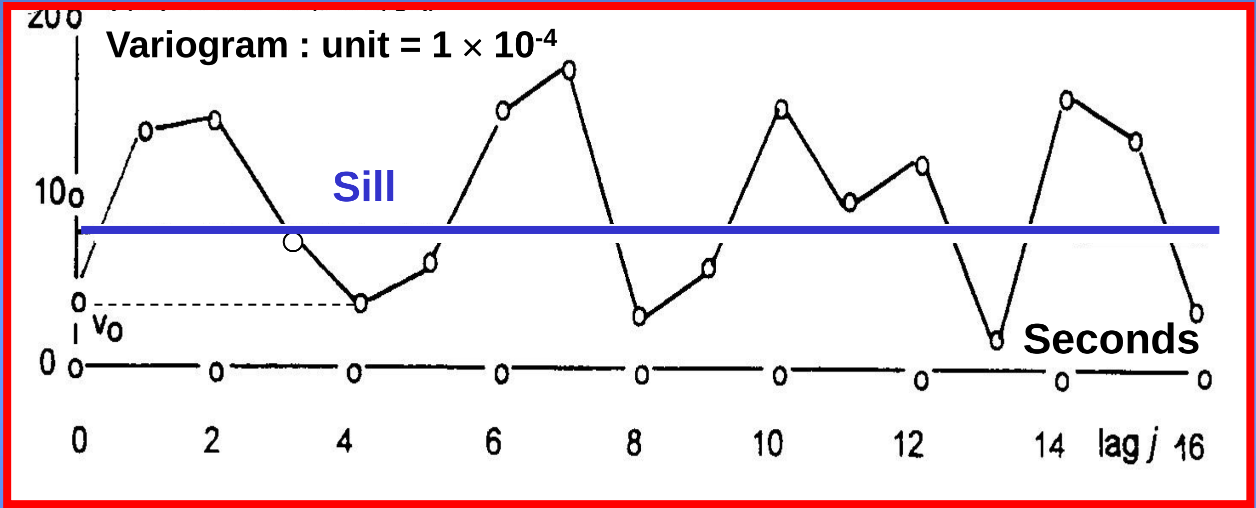
Sixth Example : Output of a bed-blending, fed to a cement kiln. Variogram of CaO %



Same material as in fifth example. Same cyclic pattern with same period. Correlation between the different components.

EXAMPLES OF VARIOGRAMS

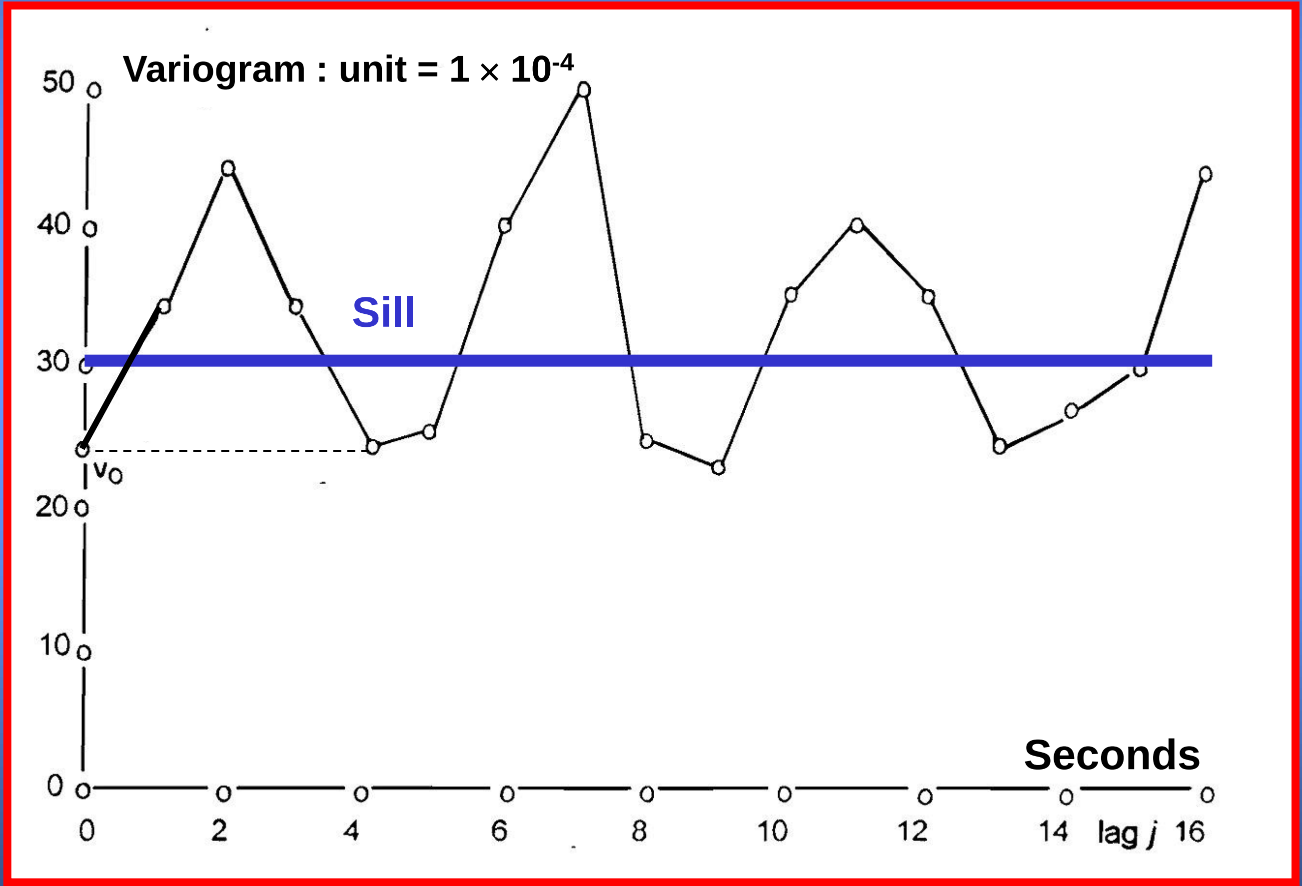
Seventh Example : Output of a bed-blending fed to a cement kiln. Variogram of Fe_2O_3



Same material as above. Same cyclic pattern with same period. Functioning of paddle chain conveyor. No practical impact.

EXAMPLES OF VARIOGRAMS

Eighth Example : Output of a bed-blending fed to a cement kiln. Variogram of Al_2O_3



Same cyclic pattern as above.

FROM THE MODEL VARIOGRAM v(j) TO THE VARIANCE σ^2(PSE)

To derive the variance σ^2**(PSE)** from the variogram **v(j)**, we must introduce several mathematical « bridges ». Those are the…

- **AUXILIARY FUNCTIONS w(j)** and **w'(j),** single / double integral means of **v(j).**
- **ERROR-GENERATING FUNCTIONS**

W_{SY} (j), W_{ST} (j) and **W_{RA} (j),** which take the « point selection mode » into account.

THE AUXILIARY FUNCTIONS S(j) and w(j) OF THE VARIOGRAM v(j)

The single and double integrals of v(j) are required in our computations. We define :

- The **single integral S(j)** of **v(j),**
- The **single integral mean w(j)** of **S(j).**

$$w(j) \equiv \frac{1}{j} S(j) \equiv \frac{1}{j} \int_0^j v(j')\, dj'$$

THE AUXILIARY FUNCTIONS S'(j) and w'(j) OF THE VARIOGRAM v(j)

We also define :

- The **double integral S'(j)** of **v(j)**
- The **double integral mean w'(j)** of **S'(j)**

$$w'(j) \equiv \frac{1}{j} S'(j) \equiv \frac{2}{j^2} \int_0^{j} dj' \int_0^{j'} v(j'')\, dj''$$

THE THREE « REFERENCE » POINT SELECTION MODES

Out of an infinity of possible selection modes we shall retain the following …

- **SY** : systematic (uniform interval T_{SY}),

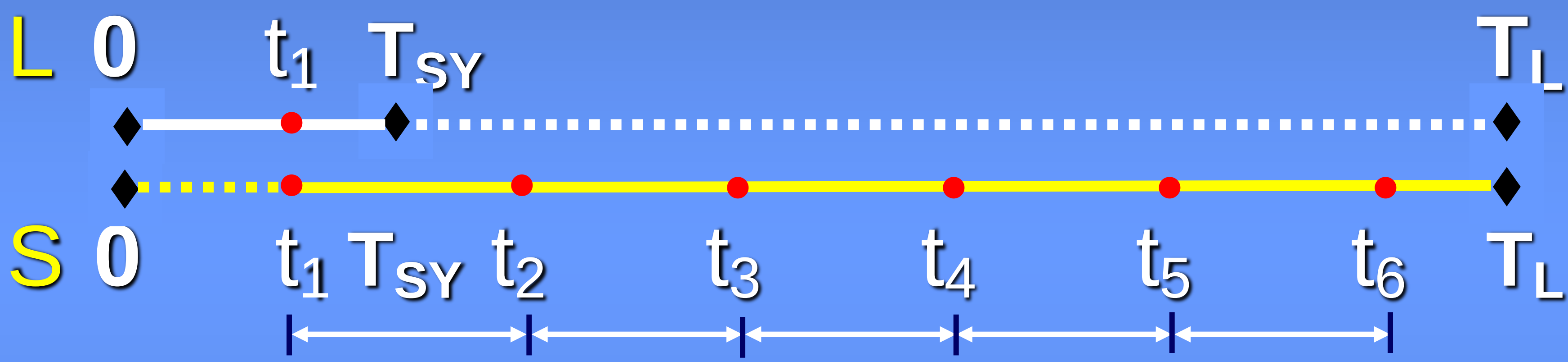
  $t_q \equiv t_1 + (q-1)\,T_{SY}$ ♦ $t_1 \equiv ran\,[\,0 < t_1 \leq T_{SY}\,]$
- **ST** : stratified random (uniform strata length T_{ST}),

  $t_q \equiv (q-1)\,T_{ST} + t'_q$ ♦ $t'_q \equiv ran\,[\,0 < t'_q \leq T_{ST}\,]$
- **RA** : random (Q random increments in L)

  $t_q \equiv ran\,[\,0 < t_q \leq T_L\,]$

- **SY** : systematic with random positioning of first increment and uniform interval T_{SY}

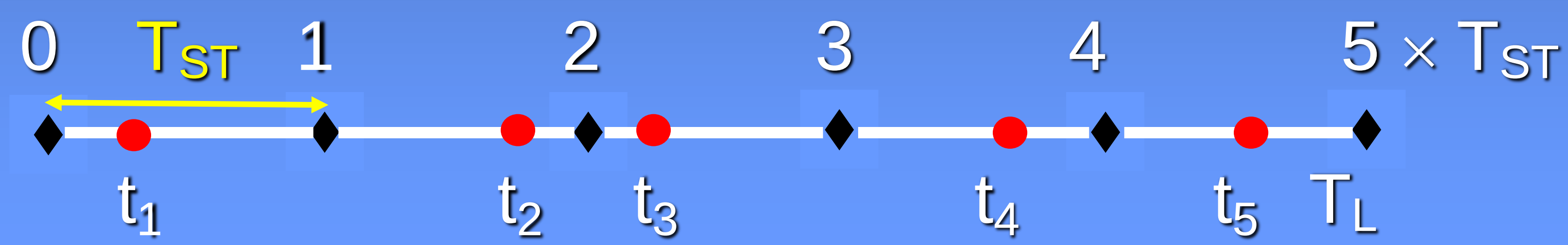
$$t_q \equiv t_1 + (q-1)\,T_{SY} \quad \blacklozenge \quad t_1 \equiv \text{ran}\,[\,0 < t_1 \le T_{SY}\,]$$



SY : most common selection mode. Easy to implement. Only shortcoming : risk of a high variance **when sampling periodic functions,** more frequent than believed.

- **ST** : stratified random (equal strata length T_{ST}), random positioning in each stratum

$$t_q \equiv (q-1)\,T_{ST} + t'_q \;\blacklozenge\; t'_q \equiv \mathrm{ran}\,[\,0 < t'_q \leq T_{ST}\,]$$



- **RA** : random (Q increments selected at random between 0 and T_L)

$$t_q \equiv \mathrm{ran}\,[\,0 < t_q \leq T_L\,]$$

- **ST** : stratified random justified in one case only : the sampling of periodic functions. In this case there is a risk when implementing a systematic selection : if the interval T_{SY} is a multiple of the period P of the function, the same point of the curve is selected. A Q-increment sample brings no more information than a one-increment sample.The variance is then multiplied by Q which often is a large number.

Grade functions with a periodic component are much more frequent than usually believed. A number of mechanical devices such as crushing and grinding circuits, **flow-rate regulating systems**, centrifugal sand or slime pumps operate in a pulsated way with a more or less uniform period.

- **RA** : Never better than **SY** or **ST**. Equivalent to assimilating a series with a population. Shows inadequacy of standards !

EGF ≡ ERROR GENERATING FUNCTIONS $W_{SY}(j)$ ♦ $W_{ST}(j)$ ♦ $W_{RA}(j)$

- **SY** : Systematic **EGF**

$$W_{SY}(j) \equiv 2\,w(j/2) - w'(j)$$

- **ST** : Stratified random **EGF**

$$W_{ST}(j) \equiv w'(j)$$

- **RA** : Random **EGF**

$$W_{RA}(j) \equiv \sigma^2(h_q) \equiv DH_L \equiv W_{RA} \equiv \text{constant}$$

The « Error-Generating Functions » of a series are the one-dimensional equivalents of the variance of a zero-dimensional population.

In other words, in both cases, the sampling variance is expressed as the **EGF** or the variance divided by the number **Q** of increments.

- **1-DIMENSION** : σ^2**(PSE)** $\equiv$ **W** $_{**}$ $\div$ **Q**
- **0-DIMENSION** : σ^2**(TSE)** $\equiv \sigma^2 \div$ **Q**

VARIANCE $\sigma^2(PSE)$ OF THE POINT SELECTION ERROR PSE

- **SY** : Systematic with interval T_{SY}

$$\sigma^2(PSE)_{SY} \equiv W_{SY}(T_{SY}) \div Q$$

- **ST** : Stratified random with strata length T_{ST}

$$\sigma^2(PSE)_{ST} \equiv W_{ST}(T_{ST}) \div Q$$

- **RA** : Random selection of **Q** increments

$$\sigma^2(PSE)_{RA} \equiv W_{RA} \div Q$$