

Why analysis needs the Theory of Sampling (TOS) – the importance of the 'before analysis' domain

Kim H. Esbensen

KHE Consulting, professor
Copenhagen, Denmark

WHY analysis needs the Theory of Sampling (TOS)

- What's behind 'Data Quality'

- There is an important past for analytical results ...
- The past has an impact on the total Measurement Uncertainty (MU_{total})
- This also impacts significantly on '**data quality**' ...

Always!!!

$$\text{Uncertainty} \leftarrow \text{TSE} + \text{TAE}$$

Total Analytical Error = TAE

Analytical result = "Data" +/- "Uncertainty" _(TAE)

Analysis: **ONLY** a **very small** fraction of a lot!

Sampling errors contribute with a **dominating** uncertainty, can be 10/25/50 X larger than TAE!

Total Sampling Error = TSE

Today's take-home lessons

- **The analytical aliquot** is the physical manifestation of transgressing the boundary from the *before analysis* (sampling) domain to the domain of analysis
- **The analytical aliquot** is the only, *miniscule* portion of the original target material that is actually analysed

... It is all about the analytical aliquot ...

- It is only possible to guarantee representativity of the analytical aliquot, and thus of the analytical results w.r.t. the **original target batch/lot/process stream**
- ... by invoking the necessary sampling competence described in the **Theory of Sampling (TOS)**

Three domains behind 'data quality'

**Before
analysis
Domain**

Sampling /
subsampling
Domain

**Domain of
Analysis**

- chemical
- physical
- other
-

**Post
Analysis
Domain**

Statistics
Data analysis
Chemometrics

+

+

Practical domain objectives ...

Domain of
sampling
competence

HOW TO
minimize TSE

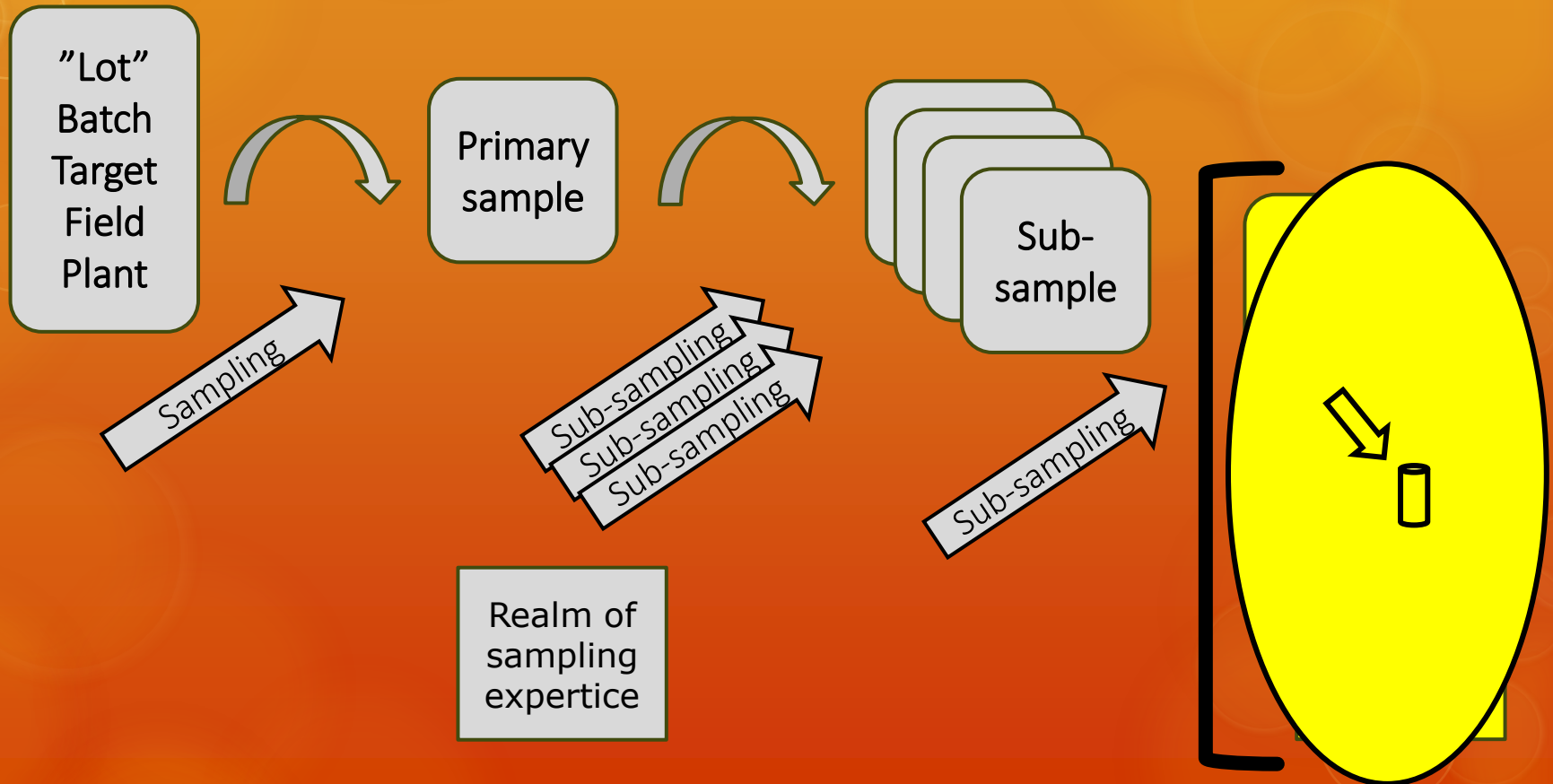
SURPRISE:

**Domain
not in
focus
today!**

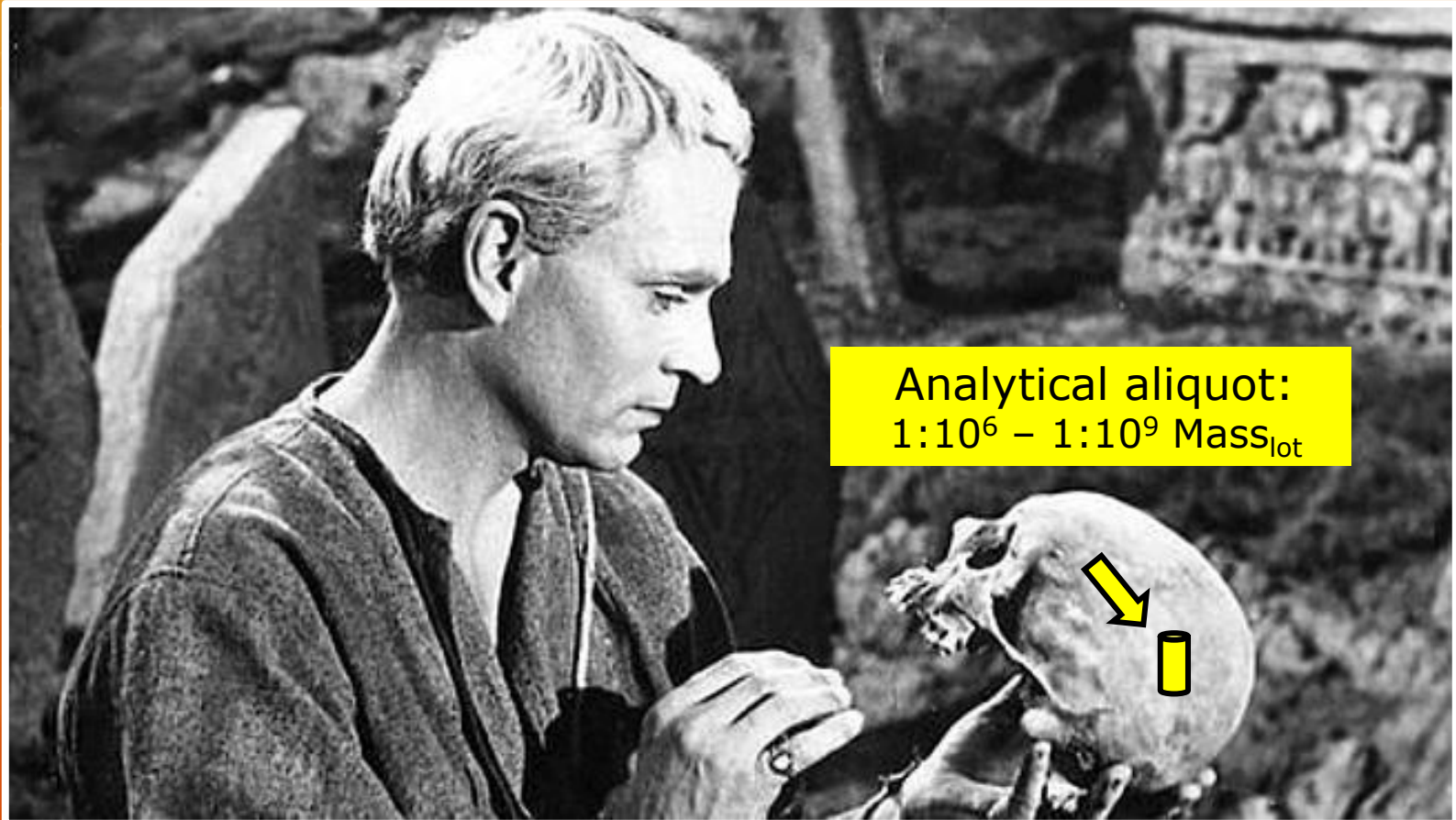
**Domain
not in
focus
today!**

Sampling/sub-sampling of

heterogeneous/lots materials



"To be, or not to be ... representative"
THAT is the question!



Analytical aliquot:
 $1:10^6 - 1:10^9 \text{ Mass}_{\text{lot}}$



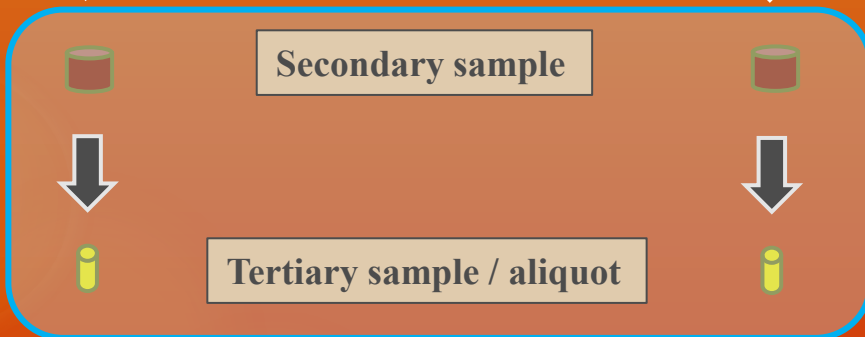
Moving lot / dynamic lot



Sampling rate
(typical)



Primary sample



Secondary sample

Tertiary sample / aliquot

Traditional laboratory domain



$10^3 - 10^6$

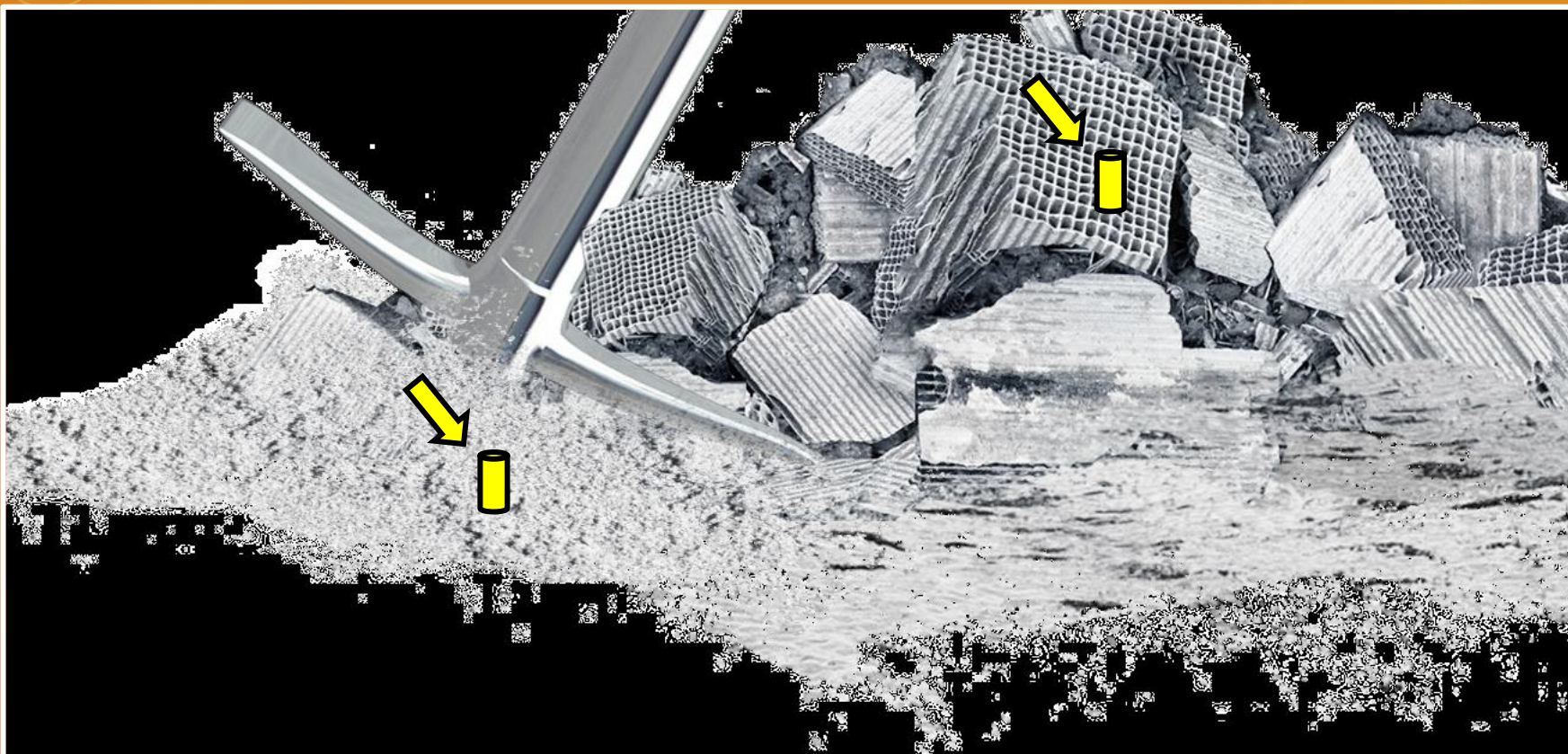
$10^1 - 10^2$

$10^1 - 10^2$



$10^3 - 10^6 - 10^9$

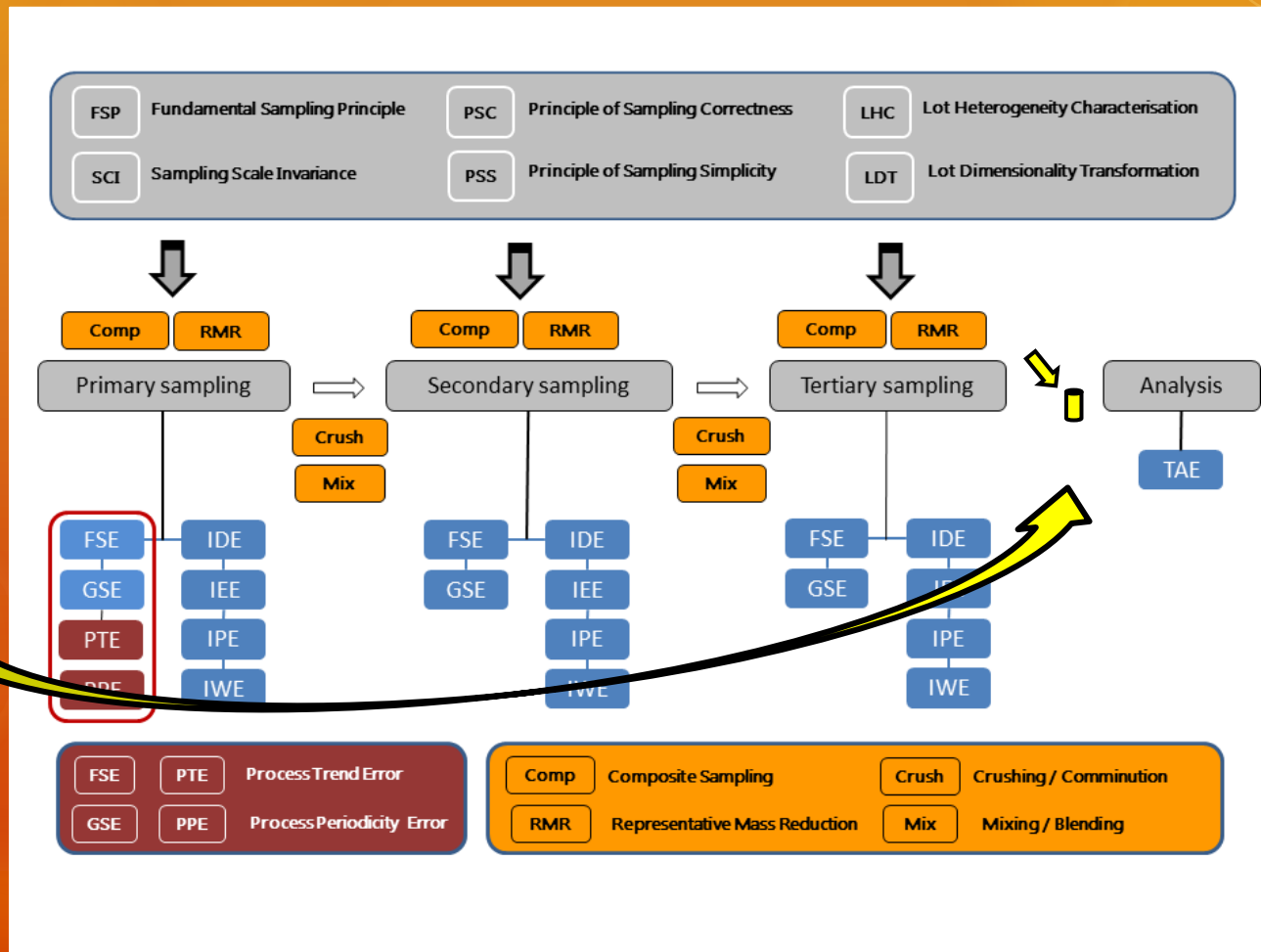
Imperative demand: Mass reduction
must be representative !!!



Theory of Sampling (TOS)

– everything in a glance

"Lot"
Batch
Target
Field
Plant



HOW TO GET A REPRESENTATIVE SAMPLE?

(Food, Feed, Pharma, Chemical, Polymer, Exploration, Environment, Mining, Metals - *process industries* o.a.)

HOW TO GET A REPRESENTATIVE ANALYTICAL ALIQUOT?

Theory of Sampling (TOS) – the critical success factor *before analysis*



Critical issue ... Scope and Focus

- Accuracy – a tribute only related to analysis of the aliquot [TAE]
- Accuracy – a tribute related to the full, compound [sampling/subsampling/analysis] process [TSE + TAE]
- Precision – an attribute only related to analysis [$\sim$ TAE]
- Precision – an attribute related the full, compound [sampling/subsampling/analysis] process [TSE + TAE]

Need a shift in focus

From: Analytical responsibility (*only*) –
To: Sampling-and-Analytical responsibilities (*both*)

"Take a sample of the lot / target material, bring it to the lab + some sample preparation – present the aliquot to the analytical instrument" - et voila" 😊😊😊😊😊😊😊😊



How to take a representative sample?





What's the common characteristic of most materials destined to be analysed?

How to take a representative sample?

WHAT characterises e.g. food/feed/agri/pharma/chemical/
polymer/mineral industry materials, and 99.50% of all others?



HETEROGENEITY

HETEROGENEITY

HETEROGENEITY





All materials met with in science, technology and industry are **heterogeneous**, it is only a matter of to which degree

Need a shift in focus

From: Analytical responsibility (*only*) –
To: Sampling-and-Analytical responsibilities (*both*)

The sampling process must be able to deal
appropriately with heterogeneity

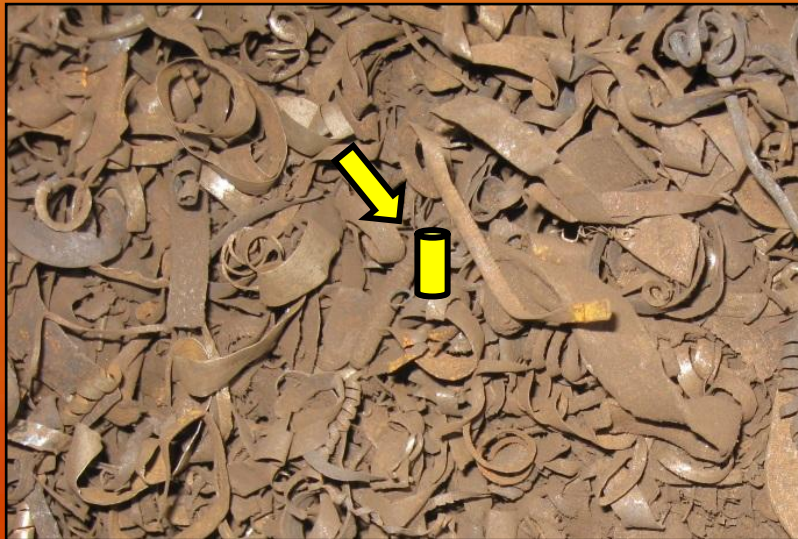


Nothing is gained by sallying forth with the aim to get the analytical aliquot - in one step ...



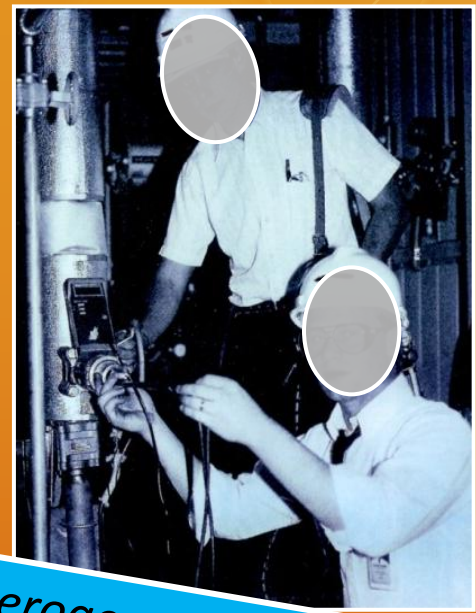


Nothing is gained by sallying forth with the aim to get the analytical aliquot in one step ...



Examples of the futility of the most often met with one-step approach:
"grab sampling"



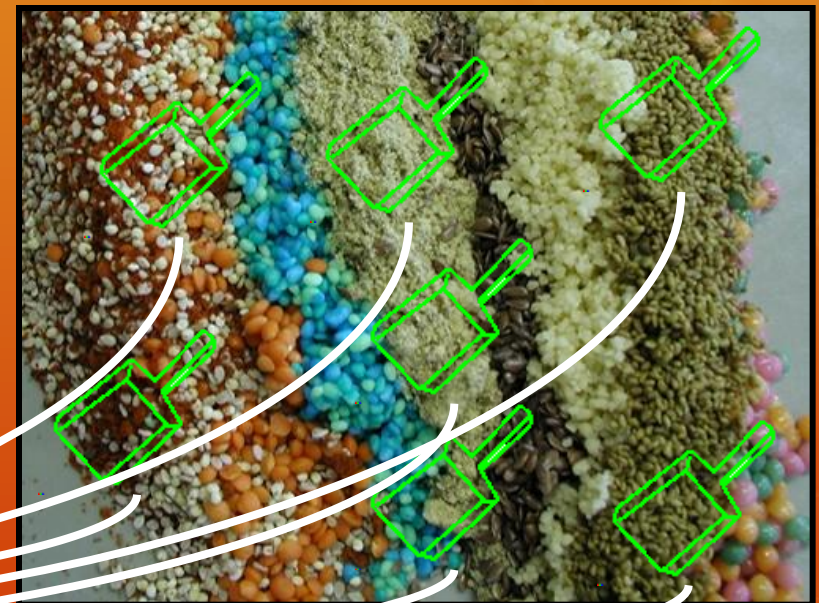


A grab sample can never be representative of a heterogeneous material

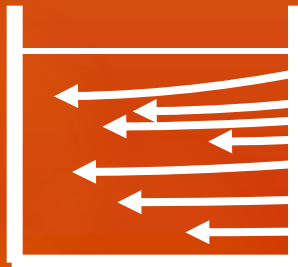
Sampling Unit Operation 1: Composite Sampling



"Grab Sampling" - "not thinking"

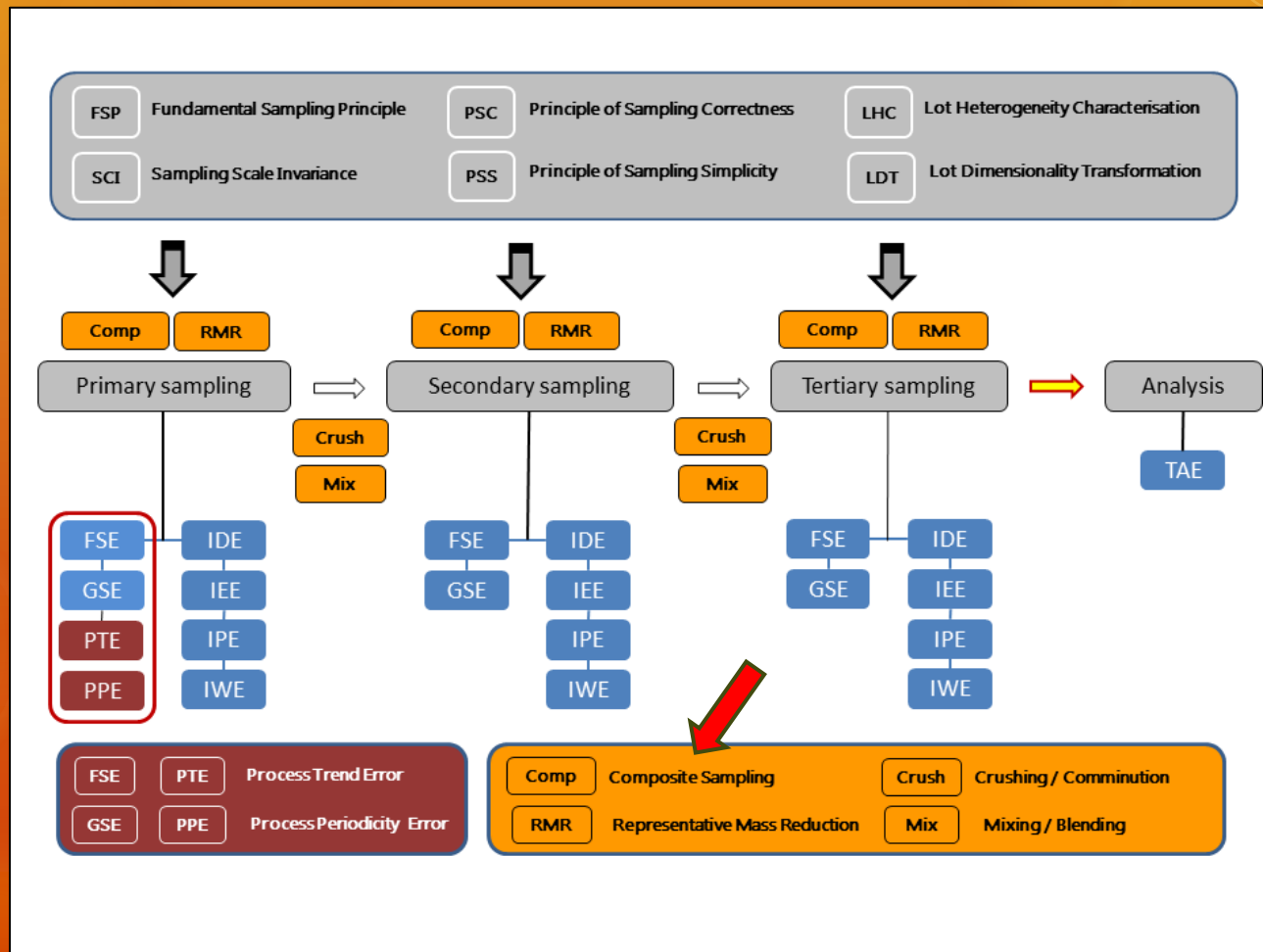


... .. Composite Sampling



Theory of Sampling (TOS)

– everything in a glance



Lot-to-aliquot: Need for representative sampling !!!

1. Sampling is never a one-shot operation (grab)
2. Sampling is always a multi-stage process (comp):

i) Primary sampling +

ii) Representative Mass-reduction

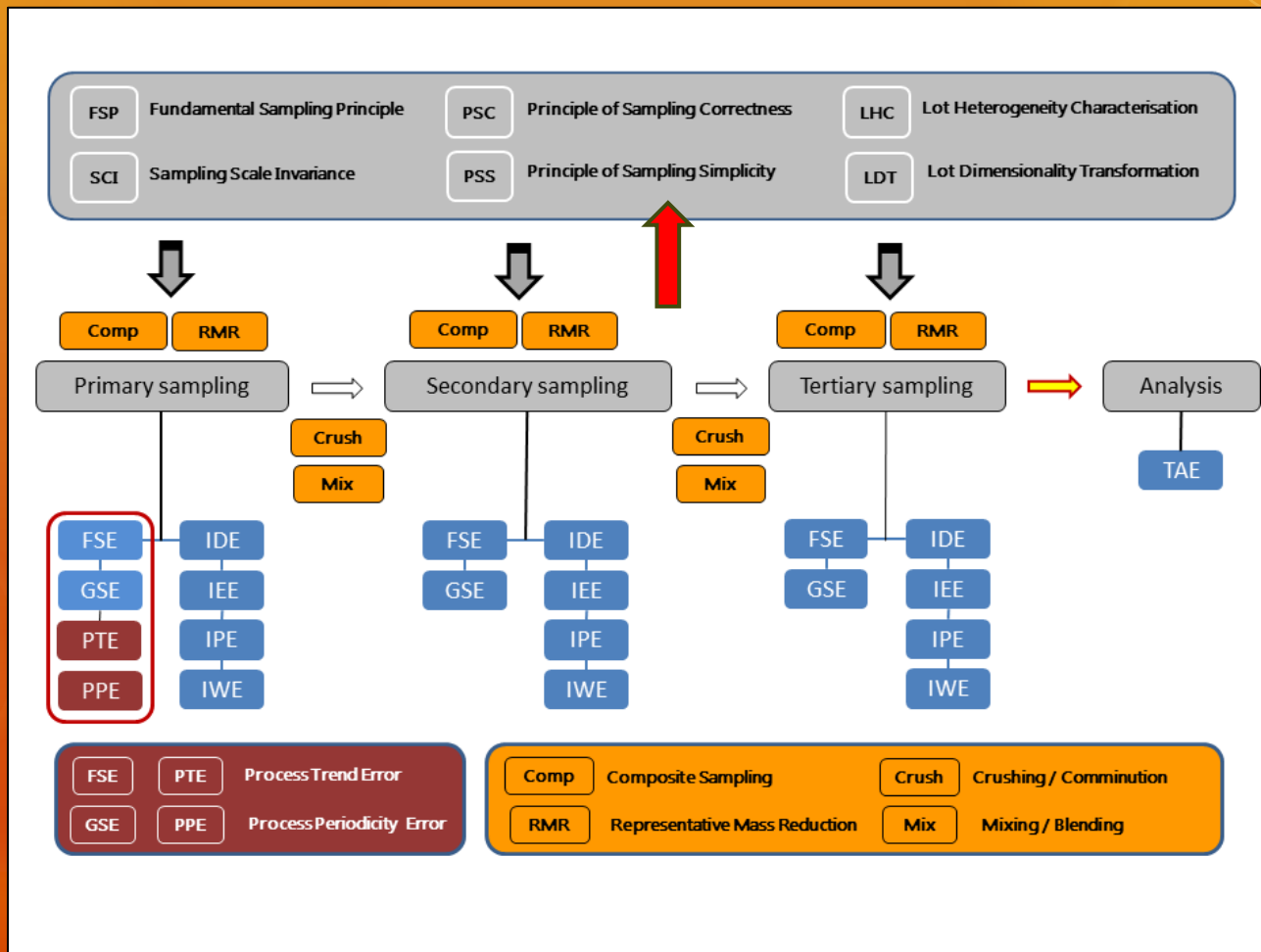


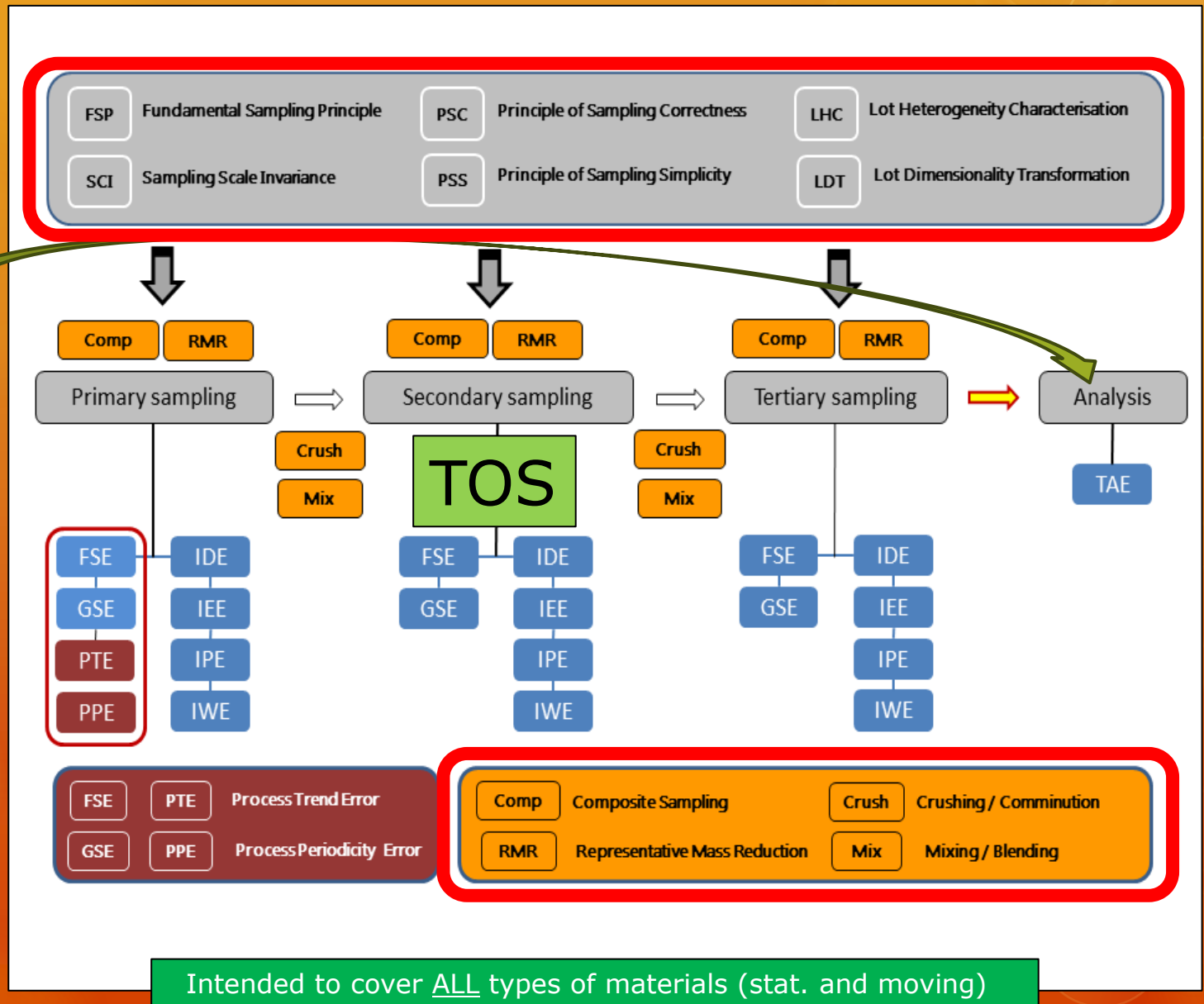
Sample splitting/division/sub-sampling ...

Principle of Sampling Simplicity (PSI)

Theory of Sampling (TOS)

– everything in a glance





Competence building / philosophy

Governing Principles (GP): 6

Sampling Unit Operations (SUO): 4(5)

- **“Once a principle has been acquired it becomes a part of you and is never lost. It can be applied to novel problems and never becomes obsolete as do all facts in a changing world (science, technology, industry, commerce, regulation)**

TOS

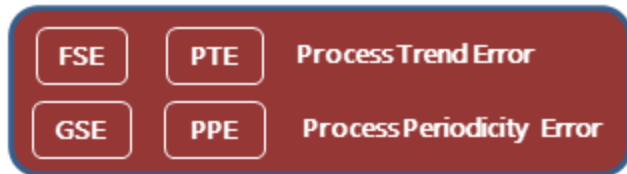
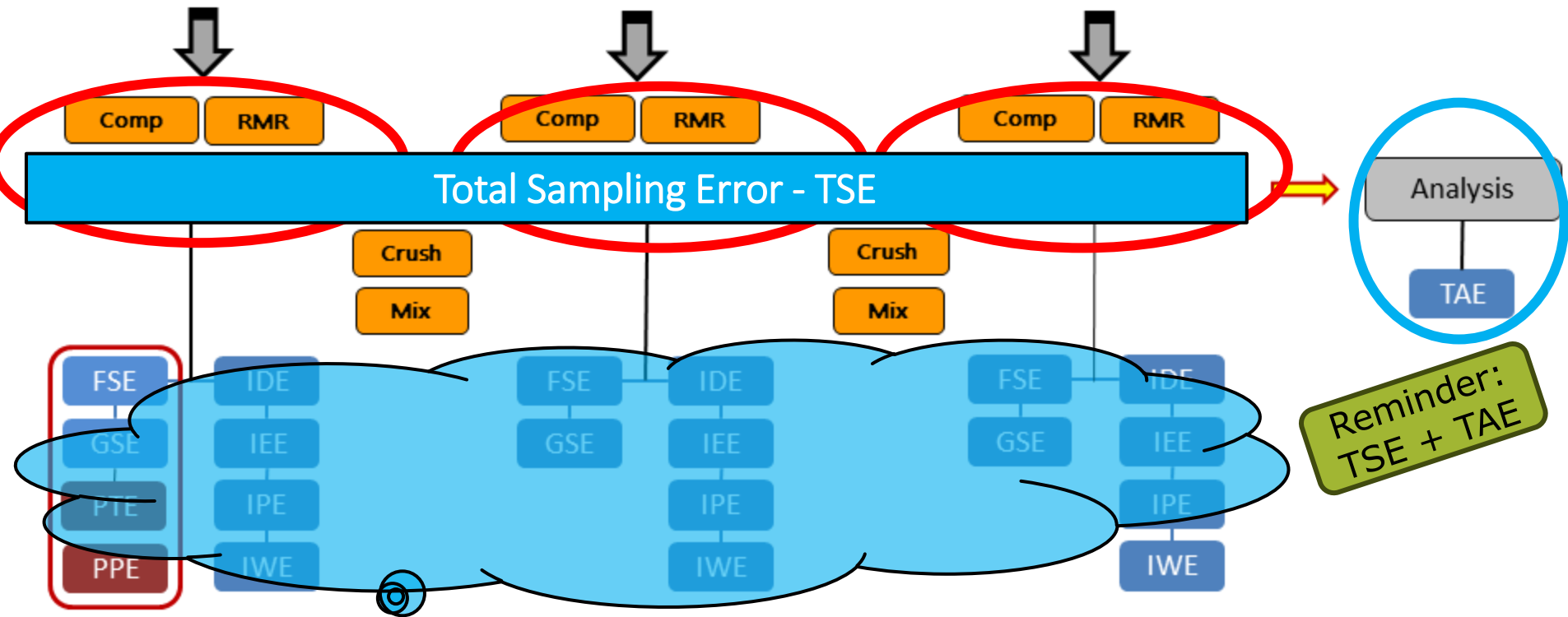
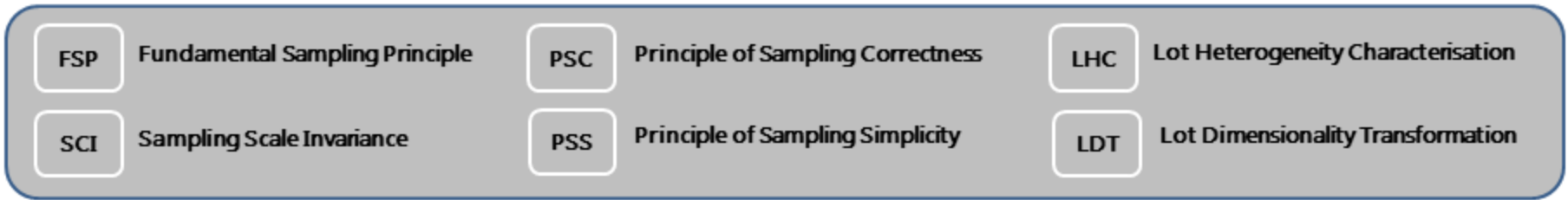
- **A well-trained mind can cope with many unforeseen problems**

TOS

- **In a rapidly changing world, no one can foresee what future problems will have to be met”**

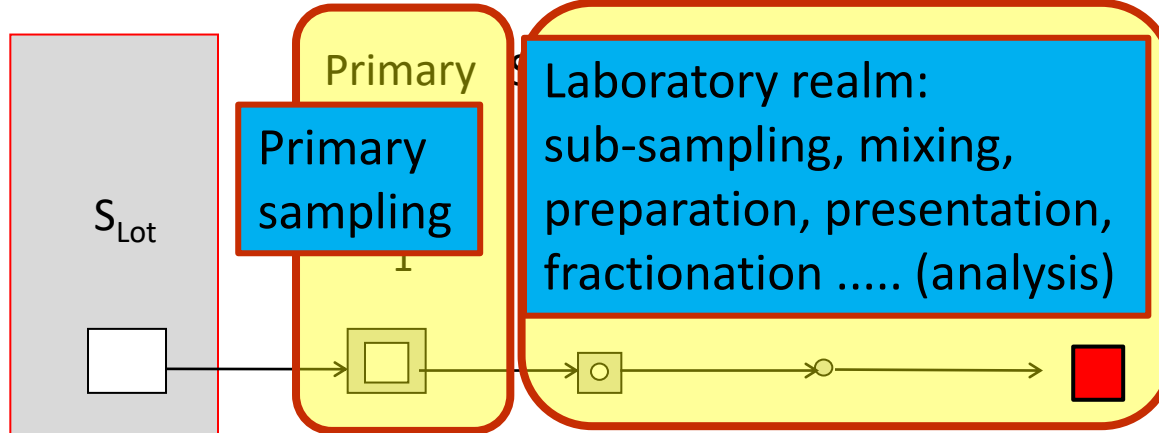
TOS

Sampling Error Management rules (SEM)



Sampling Error Management (SEM)

LOT



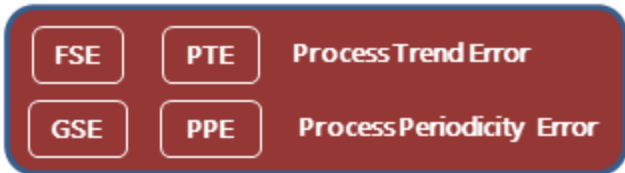
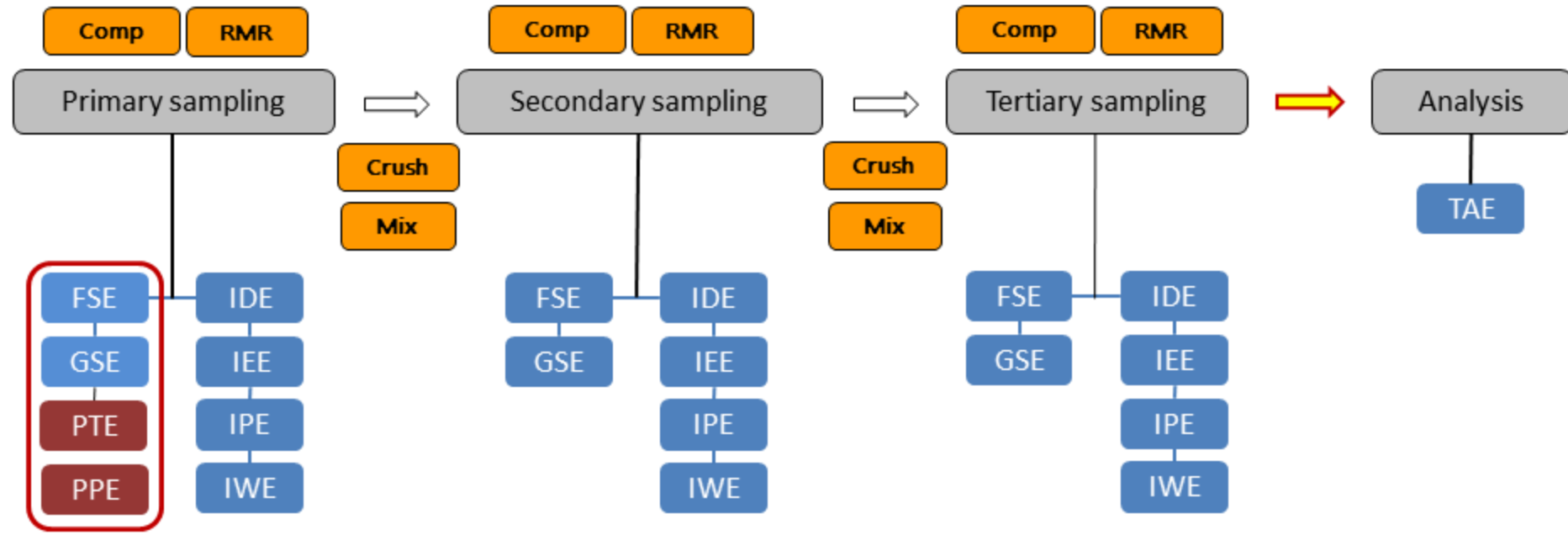
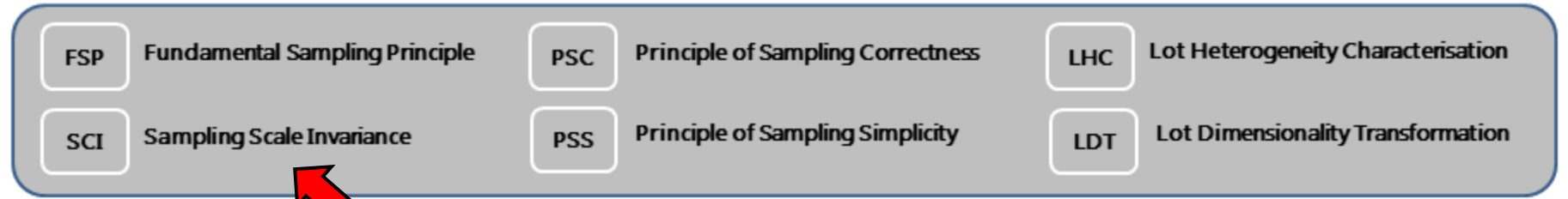
Propagation of errors: $s_x = \sqrt{\sum s_i^2}$

Example: $s_x = (55\%)^2 + (35\%)^2 + (1.5\%)^2 = (65\%)^2$

Goal: $S_x \rightarrow S_{Lot}$

Who's responsible ?

The analytical process always contains several sampling and preparation steps, but usually primary sampling dominates



... TOS in the analytical laboratory ...

Sampling equipment – must be strictly calibrated...

"UNBIASED" :-)





Sampling Scale Invariance (SSI)



Sampling in the laboratory: What's the difference w.r.t. field/plant sampling?

Field samplers and laboratory personnel → same tasks !

Theory of Sampling (TOS)

Why is TOS the critical success factor before analysis?

- for the **seller**; for the **buyer**; for the **middleman**, for the **arbiter**?
- for the **company**; **customer**; **scientist**; **technician** – for management?
- for **science**, **technology**, **industry**, **compliance**; **risk management**; **society**?

- indeed: also for the analyst!

Repræsentativ prøvetagning – Horizontal standard

Representative sampling – Horizontal standard

www.ds.dk

Horizontal – Representative Sampling

DS-3077

This standard outlines a practical, iterative, self-controlling approach with minimal complexity, based on the Theory of Sampling (TOS). The generic sampling process described and all elements involved are sufficient and necessary for the stated objective, with the consequence that no exceptions can be allowed in order to be able to document the intended sampling representativity. It is necessary to consider the full pathway from primary sampling to analytical results in order to be able to guarantee a reliable and valid analytical outcome. This standard, including normative references, annexes (and further, optional references) constitute a complete and sufficient competence basis for this purpose. The present approach will ensure appropriate levels of accuracy and precision for both primary sampling as well as for all sub-sampling procedures and mass-reduction systems at the subsequent laboratory stages before analysis.



3.43

theory of sampling

TOS

framework of *governing principles (GP)* (3.16), *sampling unit operations (SUO)* 3.41 and *sampling error management rules (SEM)* together with normative practices needed to overcome adverse effects of material heterogeneity and incorrect sampling procedures

Note 1 to entry: TOS allows to design, implement, use, and maintain proper sampling processes and equipment ensuring sampling representativeness.

Note 2 to entry: TOS history: Body of theoretical developments initiated in 1950 by the French scientist Pierre Gy, who, over the period of the following 25 years, developed a complete theory of heterogeneity, necessary and sufficient principles and operations guaranteeing representative sampling, including appropriate sampling equipment requirements (design principles, specific operational and maintenance requirements), see the Bibliography [1, 3-7, 9, 12, 13, 15, 17 and 24].

3.44

total analytical error

TAE

sum of all uncertainty effects due to a specific analytical measurement made on an analytical aliquot

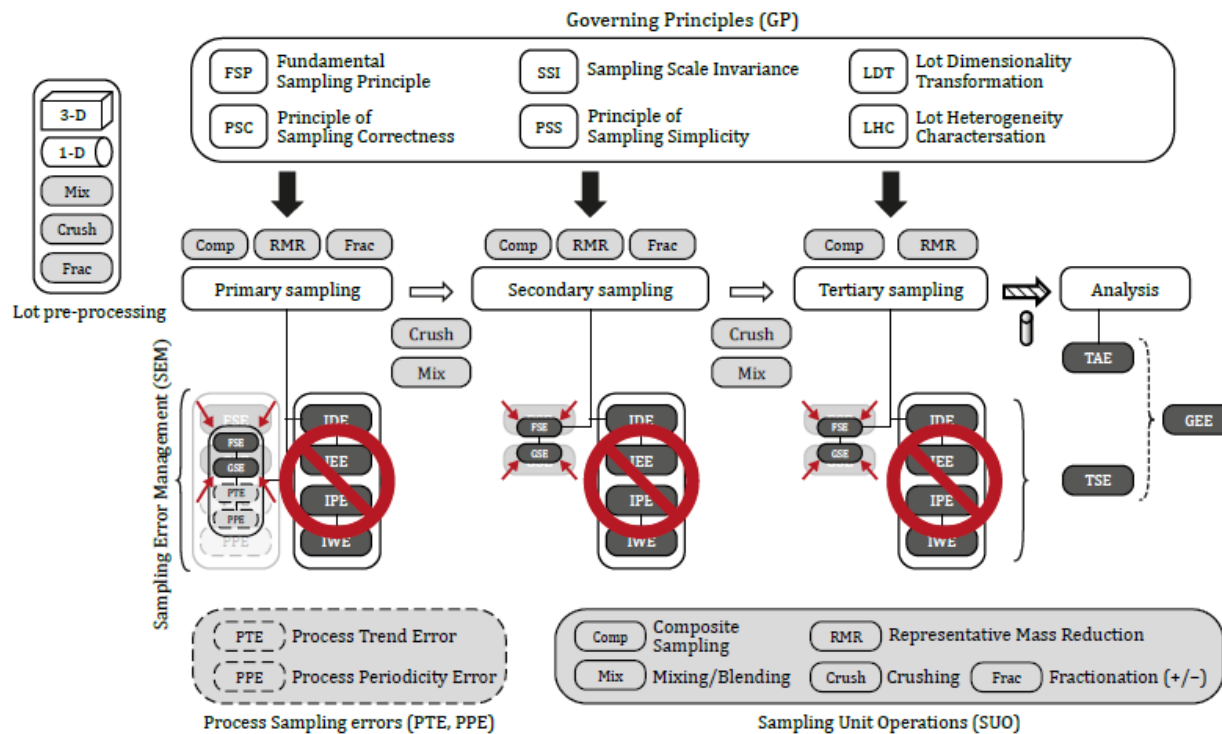
Note 1 to entry: Aliquots can be extracted as physical sub-samples, or can be intact virtual samples, e.g., a volume of a flux of matter interacting with a spectroscopic analytical instrument, or similar (PAT). TAE reflects the uncertainty effects involved in analytical method validation *per se*. TAE is specifically singled out in TOS to avoid overlapping, or confusion with respect to TSE.

3.45

total sampling error

TSE

combined uncertainty effects resulting from material extraction along the full sampling pathway from-lot-to-aliquot $TSE = ISE + CSE$, see Figure 1.



Key

- FSP Fundamental Sampling Principle (GP1)
- PSC Principle of Sampling Correctness (GP2)
- SSI Principle of Sampling Scale Invariance (GP3)
- PSS Principle of Sampling Simplicity (GP4)
- LDT Principle of Lot Dimensionality Transformation (GP5)
- LHC Principle of Lot Heterogeneity Characterisation (GP6)
- SUO 1 Composite sampling (Comp)
- SUO 2 Representative Mass Reduction (RMR)
- SUO 3 Crushing (Crush) (comminution, milling, grinding)
- SUO 4 Mixing/blending (Mix)
- SUO 5 Fractionation (Frac)
- 3-D stationary lots
- 1-D dynamic lots

Figure 11 - TOS' requirements for fit-for-purpose representativity (compare with Figure 1)

New insights ...

- Analytical results ... [the end-result of a very long, often tortuous pathway 'from-lot-to-aliquot'] form the basis for important decision-making across all of society, industry, commerce, trading, compliance regulation – decisions of great political, economic, health, resource importance.
- Analytical results *depend* on the sampling pathway ... 
- Analytical results *depend* on the sampling process:
 - representative, or
 - non-representative sampling?

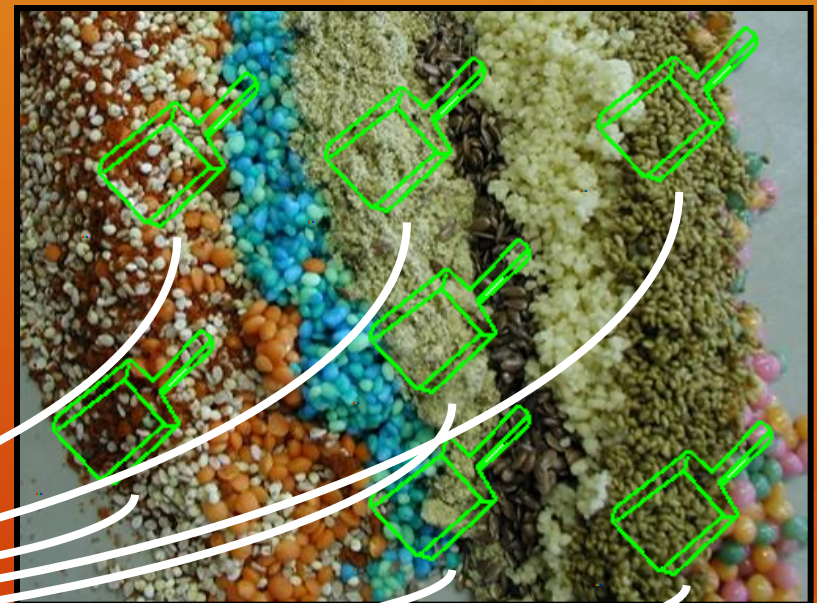
This may at first cause confusion and consternation 'in the management, legal and accounting offices' – but there is only one scientific and logical conclusion

Analytical results – depend on the sampling process

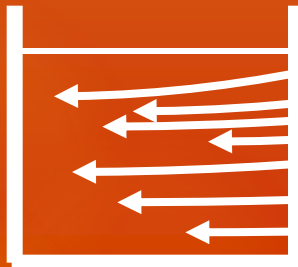


"Grab Sampling"

$$a_{S(\text{grab})} \neq a_{S(\text{comp1})} \neq a_{S(\text{comp2})}$$

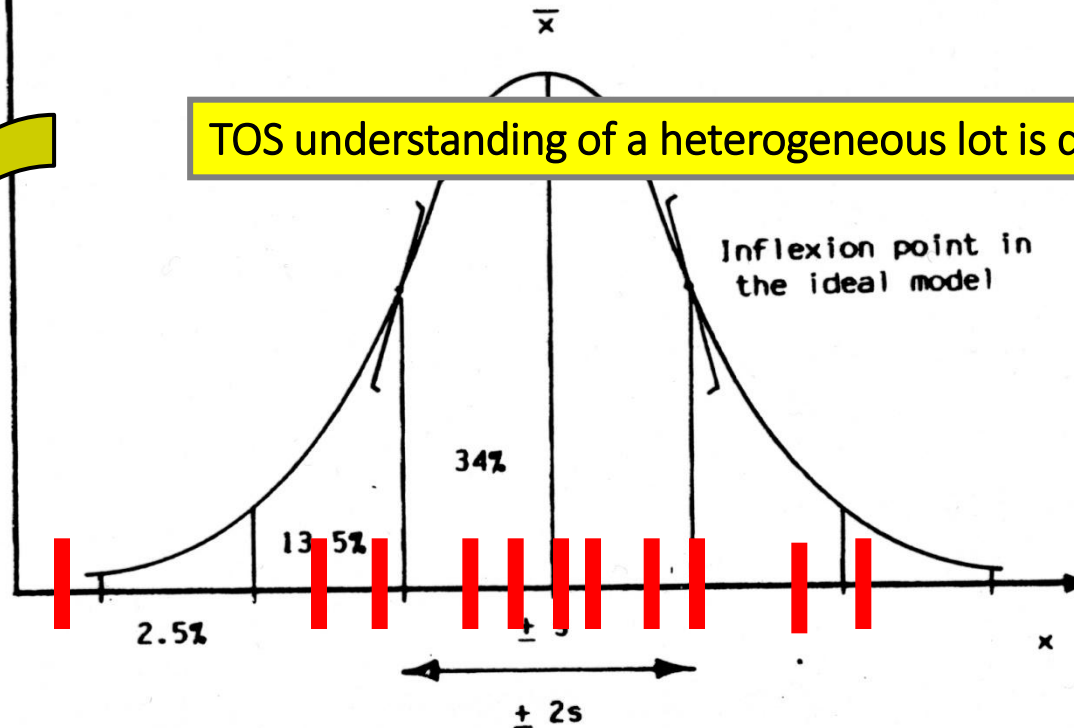


"Composite Sampling"



So, even though it is often difficult to avoid conventional statistical population thinking
NB ... population of *analytical results* NB

TOS understanding of a heterogeneous lot is different !



The (heterogeneous) lot is NOT a population of analytical results – too simple a concept !!

Understanding the heterogeneity of a material lot is not a task where trying to throw some statistics in the pot will help ...

Because:

The complete lot-to-aliquot pathway has been 100% ignored!



WHY on Earth should the wildly irregular heterogeneity of so different types of materials be of such a nature that – after sampling / sub-sampling / analysis – will the analytical results follow a Normal (Gaussian) distribution?

This is illogical

This is simply a convenient assumption because it allows to use inferential statistics to say something (apparently scientific) about the original lot

But nobody ever analysed the entire lot, so we never know ...

All materials met with in science, technology and industry are **heterogeneous**, it is only a matter of to which degree

And word (of caution) ...

- One does not 'collect' a sample
- One employs a **TOS-compliant sampling process!**
- One does not 'take' a sample ...
- One employs a **TOS-compliant sampling process!**
- Explanation: 'Samples' do not reside in the lot, ready to be 'taken', 'collected', or 'extracted'

One employs a **TOS-compliant sampling process!**

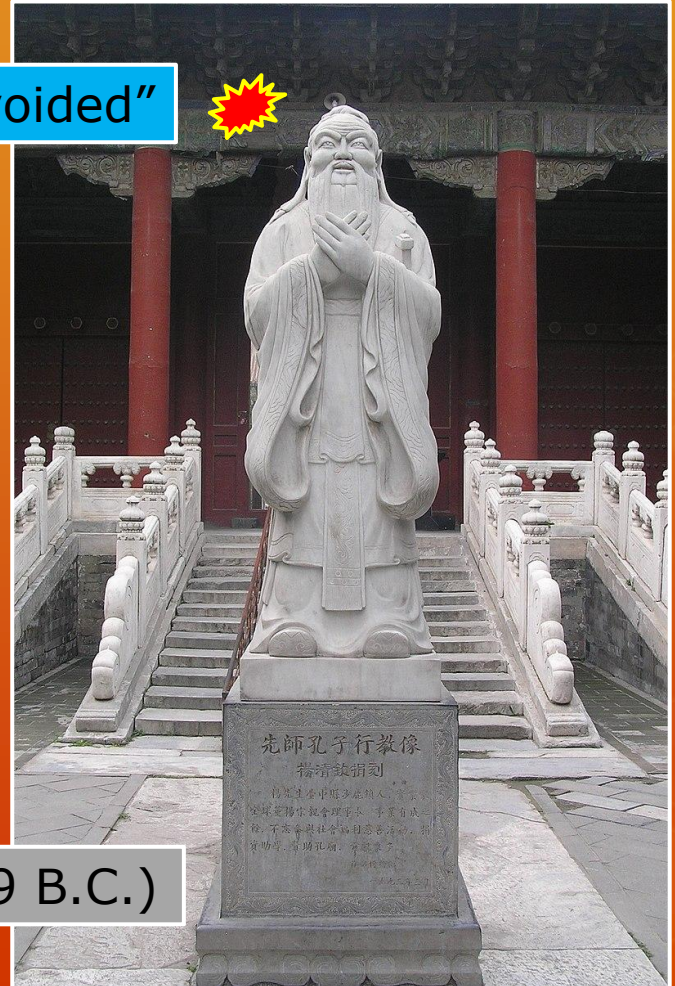
For this purpose, (also) analysts must have a minimum of TOS competence [sorry for a little hard work]

TOS – Definitions...

“Speak precisely – and wars can be avoided”



Kunfutze [Kong Futzi] (551 B.C. – 479 B.C.)



- Lot – material, size (mass), geometrical form, CH, DH, H_{grz}
- Fundamental unit – original particle, grain, fragment
- Fragment – result(s) of fragmentation of original unit(s)
- Group (of fragments) – theoretical, virtual sampling increment

'Fragments'

- Sub-sample – result of a division process/sub-process
- Increment – result of an aggregation process/sub-process

- Grab sample –

TOS

$$SE = \frac{(a_S - a_L)}{a_L}$$

- Composite sample –

'Representativity' (def)

$$r^2(SE) = m^2(SE) + s^2(SE)$$

- Sample – result of a representative sampling process (grab, comp.)
- Specimen – result of a non-representative sampling process

CRITICAL UNDERSTANDING

It is not possible to ascertain whether a sample is representative, or not – from any characterisation or evidence based on the sample itself !!!

It is only possible to demonstrate whether a *specific sampling process is* representative, or not ...

... a specific sampling process



Sampling expert's job:

Audit the **sampling process(es)** – TOS compliance, or not ...

- - - - Just a few words on a topic that causes an undue amount of confusion!

"Replicates"



Theory and Practice of Representative Sampling:

Duplication / Replication

Enormous confusion in the scientific literature !!

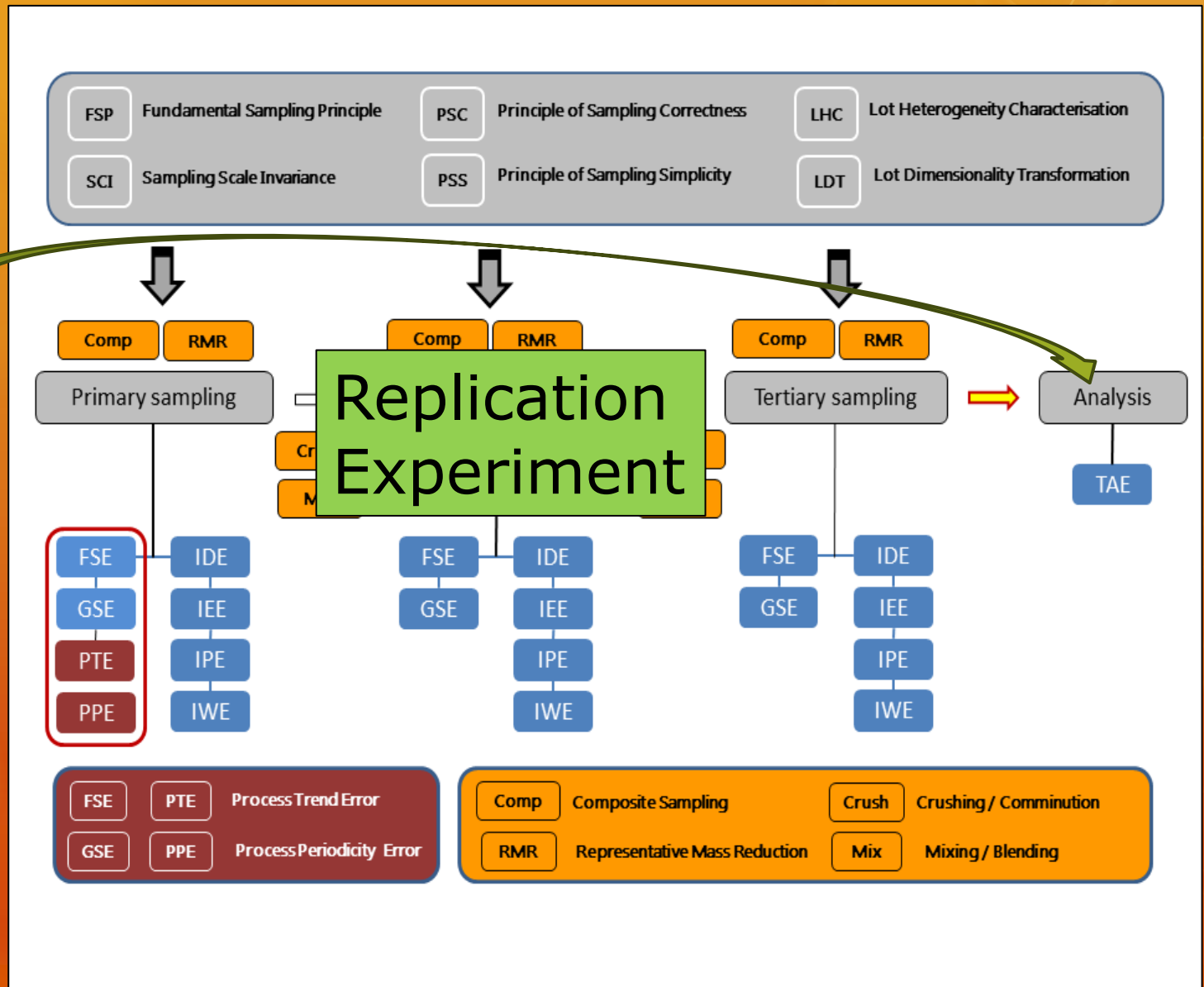
... Duplicate analysis / Duplicate analyses

... Duplicate sampling / Duplicate samples

... Replicate sampling / Replicate samples

... Replication from which sampling stage ... ???

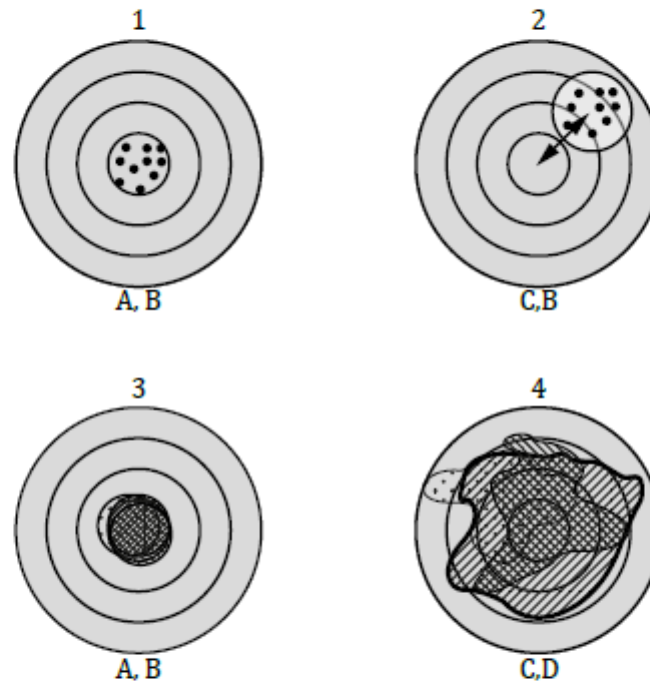
TOS



The **sampling bias** –is not (at all) like the analytical bias. Fundamentally different !!

A systematic overview of issues that distinguish the sampling bias from the analytical bias starts with acknowledging the dual set of accuracy & precision uncertainties:

- analytical accuracy and precision
- sampling accuracy and precision

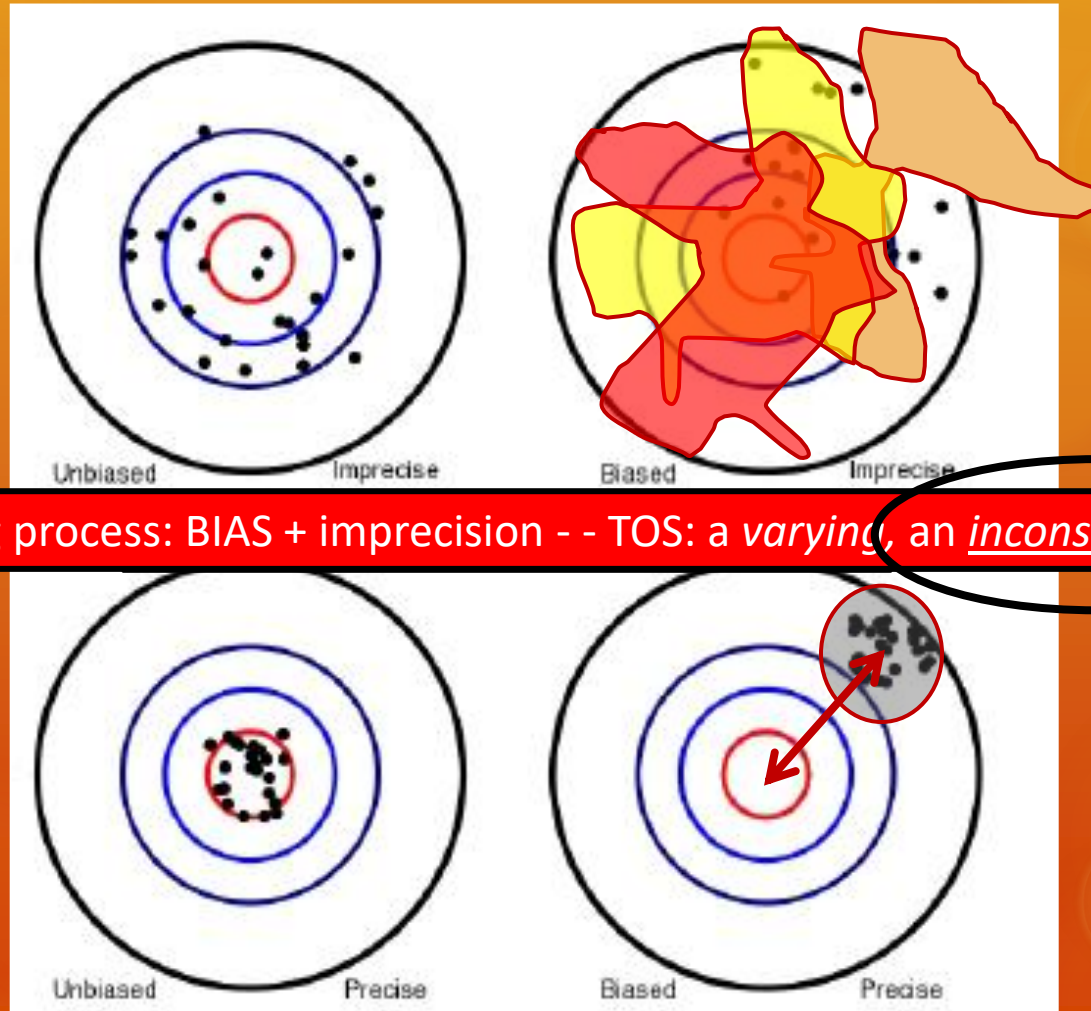


Key

- 1, 2 characteristics of the analytical bias
- 3, 4 characteristics of the sampling bias (renderings of RE envelopes)
- A accurate
- B precise
- C biased, inaccurate
- D biased, imprecise

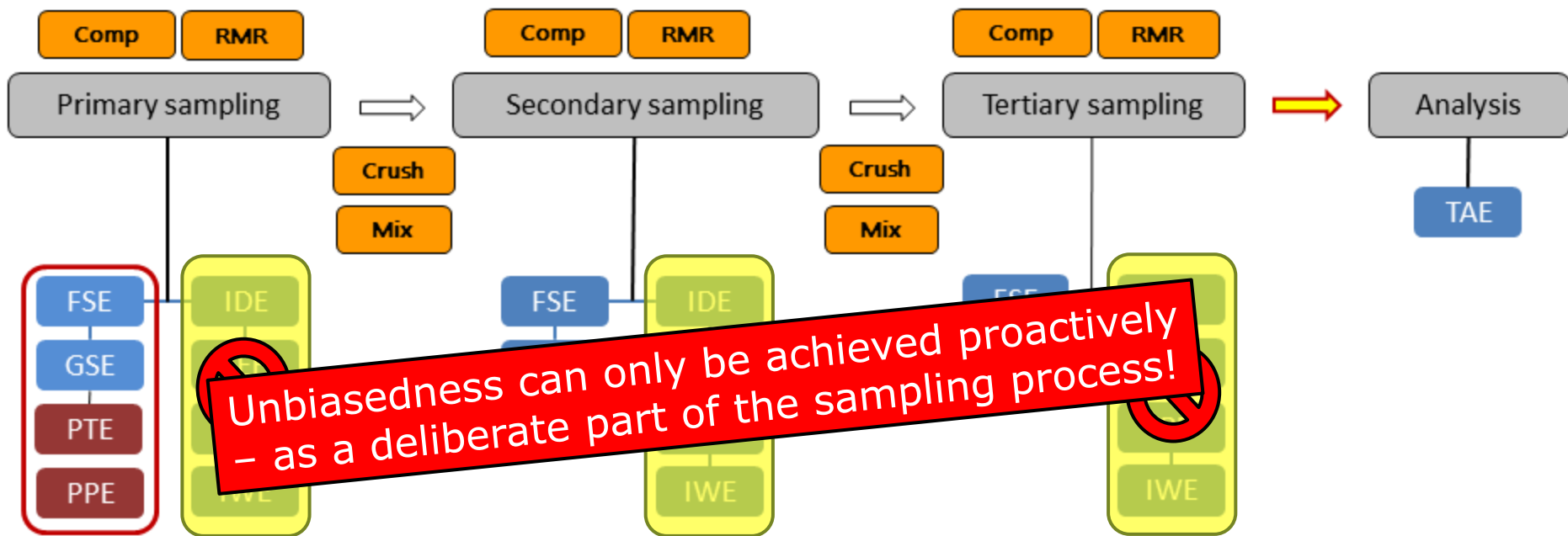
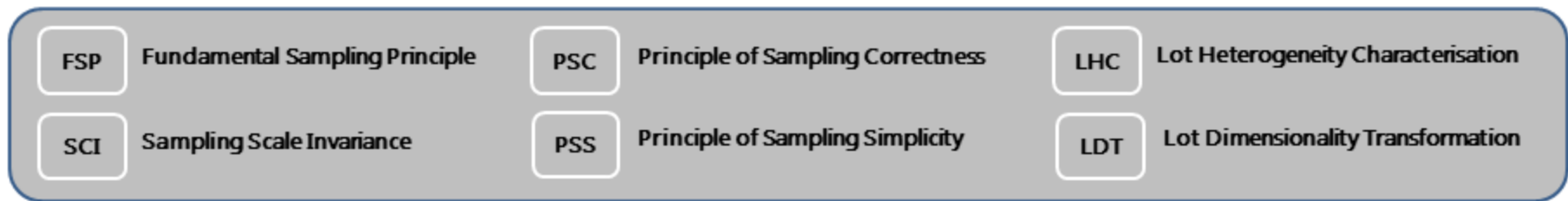
Figure 12 - Characteristics of the analytical bias versus the sampling bias

Critical distinction between analysis (*sensu strictu*) –
and sampling_plus_analysis! TOS' unique understanding!

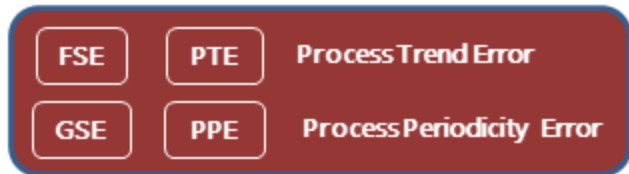


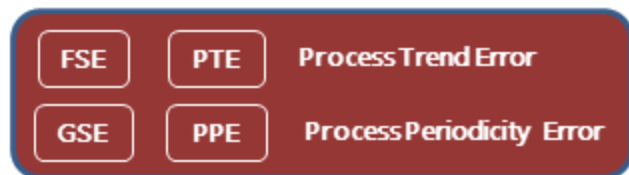
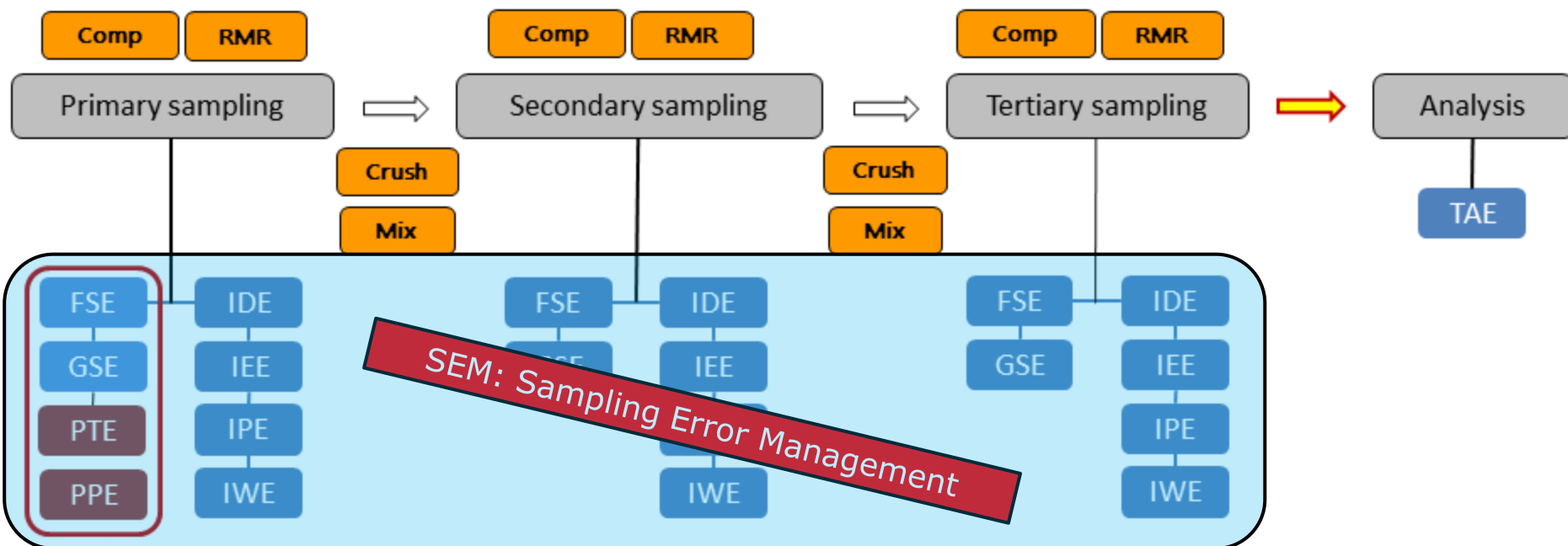
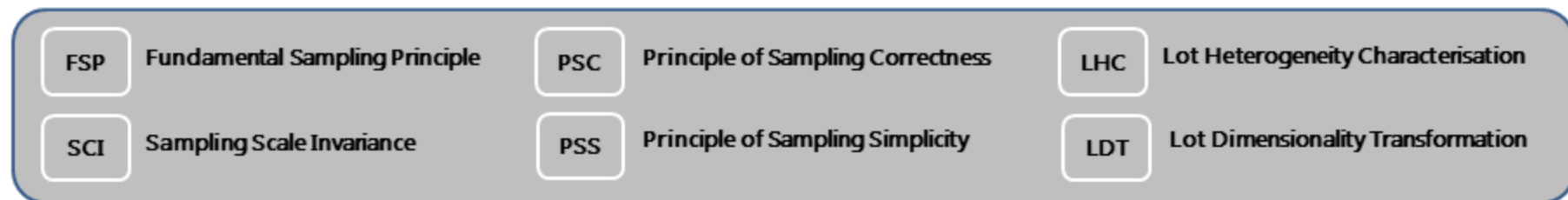
Sampling process: BIAS + imprecision - - TOS: a *varying*, an inconstant bias!

Analysis: bias + imprecision (statistical concept: a constant bias)



Unbiasedness can only be achieved proactively
 – as a deliberate part of the sampling process!



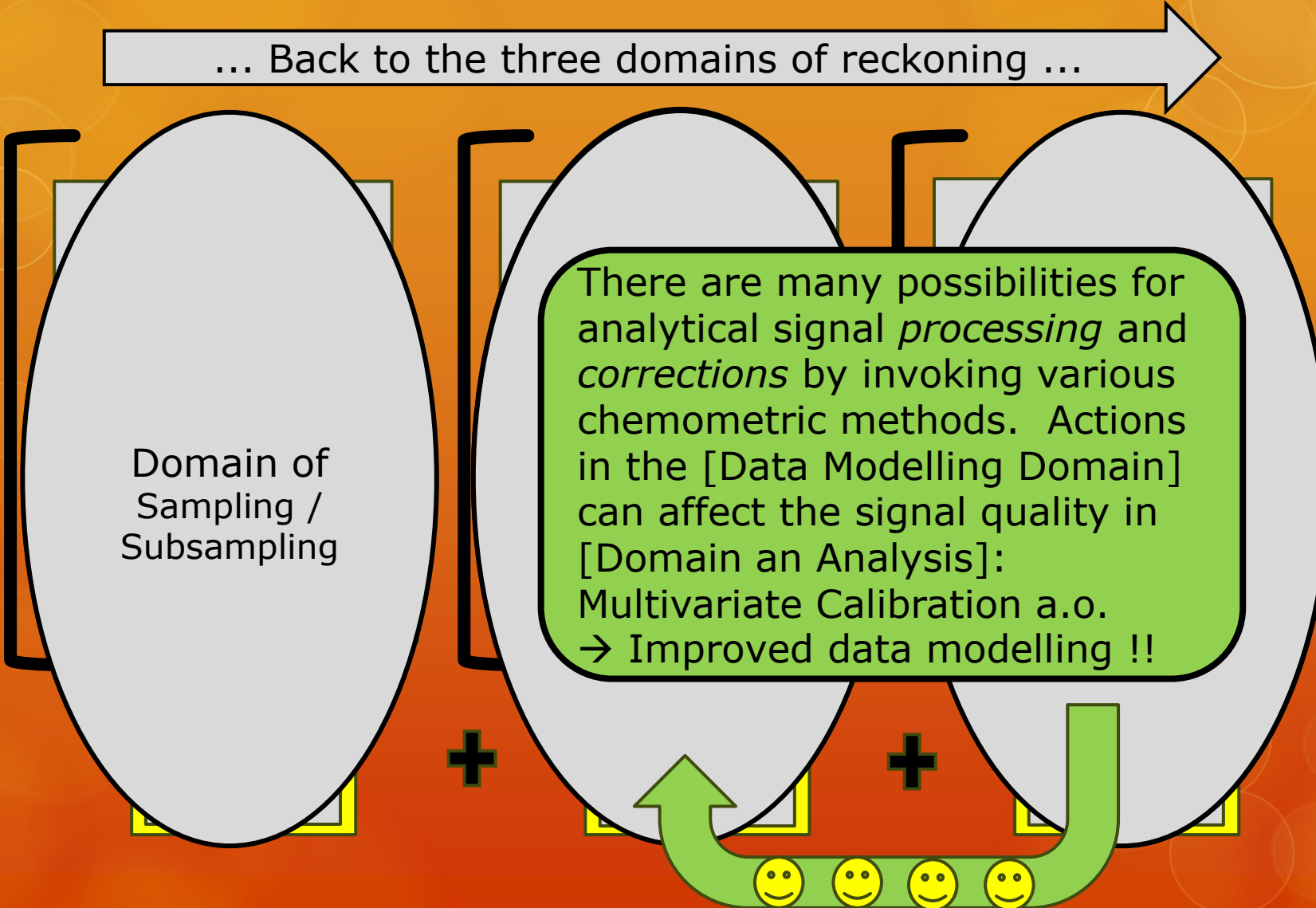


Repræsentativ prøvetagning – Horizontal standard

Representative sampling – Horizontal standard

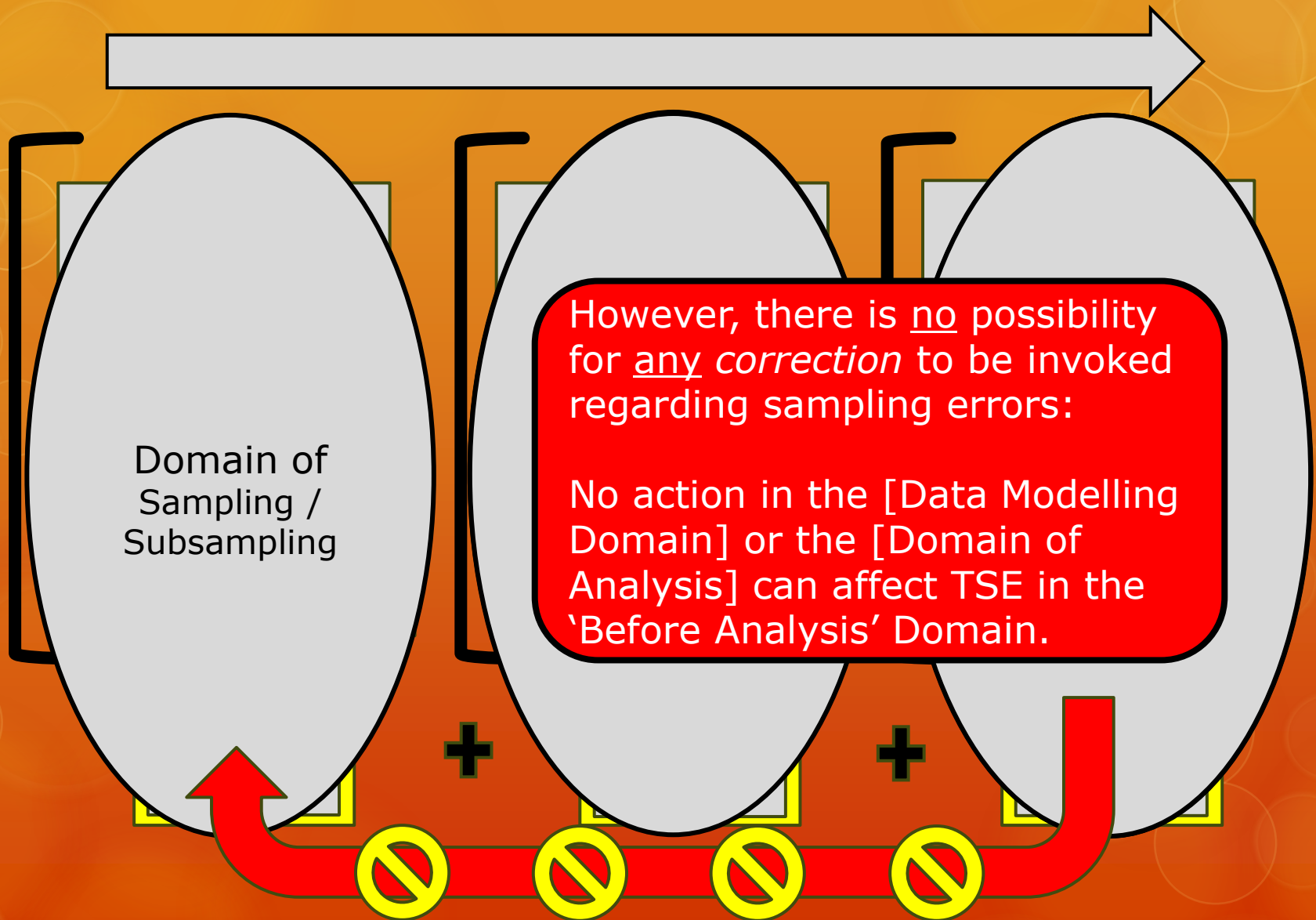


... Back to the three domains of reckoning ...



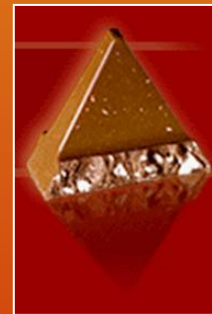
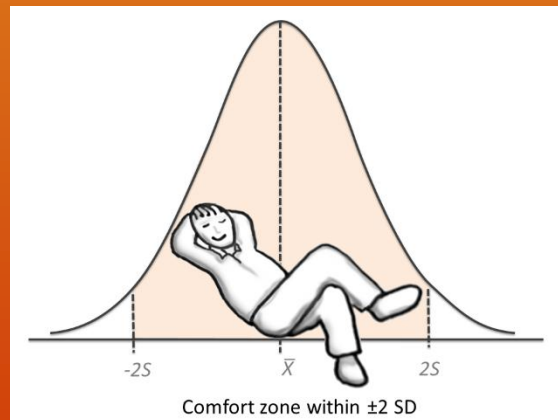
Domain of
Sampling /
Subsampling

There are many possibilities for analytical signal *processing* and *corrections* by invoking various chemometric methods. Actions in the [Data Modelling Domain] can affect the signal quality in [Domain an Analysis]:
Multivariate Calibration a.o.
→ Improved data modelling !!



THE real problem: assume normality (analytical results)

It most emphatically is **not** – heterogeneity does not follow a normal distribution (.. of analytical results)!



Sampling solutions – technology; sampling conditions; economy; complexities; risk management

Framing TOS in Risk Assessment - an outreach perspective

Claudia Paoletti & Kim H. Esbensen

Proceedings of WCSB10: TOS Forum Issue 11, 419–423 (2022)



Today's most important take-home message:
Two sentences to be inserted in all contracts,
standards, guiding documents .. Everywhere!

**"All sampling procedures invoked to
secure primary samples (as well as
all sub-sampling operations needed
to produce the analytical aliquot),
as applied by all samplers shall be
in compliance with the principles of
representative sampling as laid out
by the Theory of Sampling (TOS).**

**All sampling procedures shall be
adequately and fully documented."**

The TOS imperative !!!

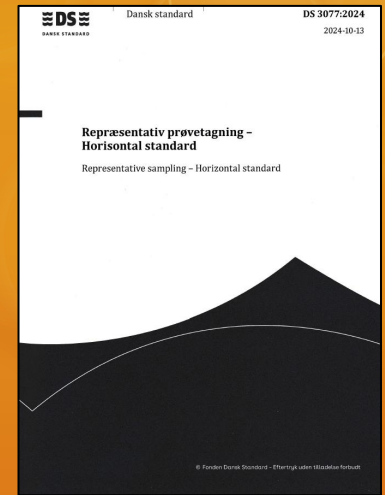


WHAT can be accomplished in 30 minutes?

More than enough!

You are now well aware of **DS 3077:2024**

You can download it from here: **www.ds.dk**



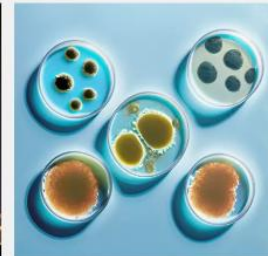
A bonanza of **Open Access** literature is available ...

1. F.ex. from International Pierre Gy Sampling Association:

<https://www.intsamp.org/>

2. Center of Sampling Competence (CSC)

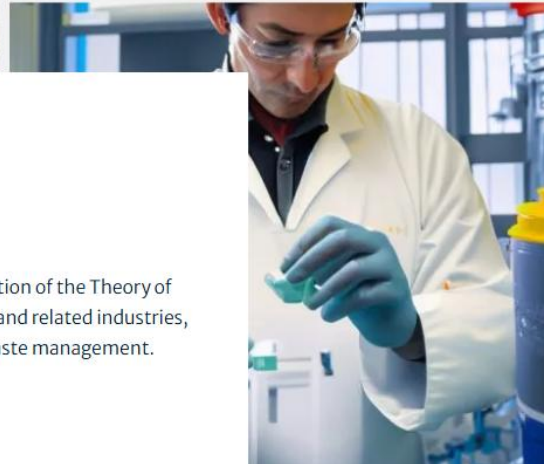
<https://www.sampling-competence.com/>



International Pierre Gy Sampling Association

The International Pierre Gy Sampling Association, IPGSA, promotes the development and application of the Theory of Sampling (TOS). TOS was founded by Pierre Gy in the 1950s to address sampling issues in mining and related industries, and since then the IPGSA has applied TOS in manufacturing, pharmaceuticals, agriculture, and waste management.

[Learn more about the IPGSA](#)





International Pierre Gy Sampling Association (IPGSA) • SST-Magazine

Sampling Science and Technology

Sampling Science and Technology (SST) is a free publication for the sampling community, providing a communications platform for all interested in the theory and practice of representative sampling and blending. SST aims both to serve the world sampling community and to disseminate awareness of proper sampling practices to all interested parties in science, technology, industry, and commerce, as well as to supervising and regulating authorities.

SST is the official communication platform of the International Pierre Gy Sampling Society (IPGSA). SST is offered as a forum for scientific and technological interaction among all members of the world sampling community and beyond. The aim of SST is to share knowledge on the science and practice of sampling and blending. As such, SST welcomes any-and-all research, practical perspectives, exemplary case histories, as well as the occasional theoretical article aimed at all sampling practitioners.

Read the latest issue of SST

Read the articles in the current issue or visit sst-magazine.info to learn more about the magazine and back issues. There you will also find the complete TOS forum archive.

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Introduction to Sampling

Your interactive Guide to the Theory and Practice of Sampling, TOS

Are you in need of reliable data in any sectors e.g., mining, mineral resources, pharmaceuticals, food, feedstock, environmental science and engineering, recycling to take informed decisions? If so, you need sampling. This interactive infographic will guide and help you by presenting the basic principles and practical operations for representative sampling, which will guarantee optimal data quality and reliability.

Data is requested from a responsible Sampling Manager.

[Learn more](#)

STAKEHOLDER REQUEST

Financial resources, personnel
and equipment.

[Learn more](#)

RESOURCES



Data always includes Total
Uncertainty.

[Learn more](#)

DATA UNCERTAINTY

375 ppm
+/- 85 ppm



SIX PUBLISHING

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Introduction to the Theory and Practice of Sampling

by Kim H. Esbensen





Thank you for your attention!

Kim H. Esbensen, KHE Consulting (KHEC), Copenhagen, Denmark
Associate-, visiting-, and external professor (Norway, Puerto Rico, Quebec)